

MATH 2260 - PRACTICE FINAL EXAM

Problem 1. Let R be the region bounded by the curves: $x = 1 - y^2$ and $x = 0$.

- Find the volume of the solid obtained by revolving the region R around the y -axis.
- Find the volume of the solid obtained by revolving the region R around the x -axis.

Problem 2. Find the length of the graph of the function: $y = \frac{1}{2}x^2 + \frac{1}{4}\ln(x)$ over the interval $1 \leq x \leq 2$.

Problem 3. Use integration by parts to calculate

- $\int e^x \sin(2x) dx$
- $\int \ln^2(x) dx$

Hint: First use a substitution: $x = \tan(u)$.

Problem 4. Evaluate the following integrals:

- $\int \tan^2(x) dx$
- $\int \sin^2(2x) \cos^2(2x) dx$
- $\int \sqrt{1 - \cos(2x)} dx$

Problem 5. Evaluate the following integrals using trigonometric substitution

- $\int \frac{dx}{4+x^2}$
- $\int \frac{dx}{\sqrt{1+9x^2}}$
- $\int \frac{\sqrt{x^2-25}}{x} dx$

Problem 6. Determine whether the following improper integrals converge using the Comparison or Limit Comparison test.

a) $\int_2^{\infty} \frac{1}{\sqrt{x^4-1}} dx$

b) $\int_1^{\infty} e^{-\sqrt{x}} dx$

Hint: First do a substitution $u = \sqrt{x}$ and then use the comparison test.

Problem 7. Use the Comparison or the Limit Comparison test to determine the convergence of the following series.

a) $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}\sqrt{n^2+1}}$

b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$

Hint: First determine: $\lim_{n \rightarrow \infty} \sqrt[n]{n}$, by taking its logarithm and applying L'Hospital's rule.

Problem 8. Use the Ratio, Root or Alternative Series test to determine convergence of the following series.

a) $\sum_{n=1}^{\infty} \frac{n^{10} 10^n}{n!}$

b) $\sum_{n=2}^{\infty} \frac{2^n}{(\ln n)^{n/2}}$

c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

d) $\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n}}{\sqrt{n+\sqrt{n+1}}}$

Problem 8. Consider the triangle with vertices: $A = (1, 0, 1)$, $B = (2, 1, 0)$, $C = (1, -2, 1)$.

a) Find the length of the two sides: \overrightarrow{AB} , \overrightarrow{AC} .

b) Find the angle between the above two sides.

c) Find the distance of the vertex C from the side \overrightarrow{AB} (i.e. find the height of the triangle if the base is \overrightarrow{AB}).

d) Find the area of the triangle.