

## MATH 2260 - SOLUTIONS

**Problem 1.** Part (a) is "washers", part (b) is "cylindrical shells".

$$(a) V = \int_{-1}^1 \pi(1 - y^2)^2 dy = 2\pi(1 - 2/3 + 1/5)$$

$$(b) V = \int_0^1 2\pi y(1 - y^2) dy = \pi/2$$

**Problem 3.** You need to do two integration by parts for both parts.

$$(a) \int e^x \sin(2x) dx = (e^x \sin(2x) - 2e^x \cos(2x))/5 + C$$

$$(b) \int \ln^2(x) dx = x \ln^2(x) - 2 \int \ln(x) dx = x \ln^2(x) - 2x \ln(x) + 2x + C$$

**Problem 4.**

$$(a) \int (\sec^2(x) - 1) dx = \tan(x) - x + C$$

$$(b) \frac{1}{4} \int (1 - \cos(4x))(1 + \cos(4x)) dx = \frac{1}{8} \int (1 - \cos(8x)) dx = x/8 - \sin(8x)/64 + C$$

$$(c) \int \sqrt{2\sin^2(x)} dx = -\sqrt{2}\cos(x) + C$$

**Problem 5.**

$$(a) (x/2 = \tan(\theta)); \quad \int = \frac{1}{2} \tan^{-1}(x/2) + C$$

$$(b) (x = \frac{1}{3} \tan(\theta)); \quad \int = \frac{1}{3} \ln |3x + \sqrt{1 + 9x^2}| + C$$

$$(c) (x/5 = \sec(\theta)); \quad \int = 5 \int \tan^2(\theta) d\theta = \sqrt{x^2 - 25} - 5\sec^{-1}(x/5) + C$$

**Problem 6.**

(a) Using the Limit Comparison test:

$$\lim_{x \rightarrow \infty} (1/\sqrt{x^4 - 1})/(1/x^2) = \lim_{x \rightarrow \infty} \sqrt{1 + 1/(x^4 - 1)} = 1$$

The integral converges as

$$\int_2^{\infty} \frac{1}{x^2} dx$$

converges.

(b) Substitute:  $u = \sqrt{x}$ , that is  $x = u^2$ , so  $dx = 2udu$ . Then

$$\begin{aligned} \int_1^N e^{-\sqrt{x}} dx &= \int_1^M 2ue^{-u} du = \\ &= 2(e^{-1} - Me^{-M}) + 2 \int_1^M e^{-u} du \rightarrow 4e^{-1} \end{aligned}$$

as  $M = \sqrt{N} \rightarrow \infty$ , where the second equality is obtained by a partial integration. The improper integral obtained converges, thus the original integral converges as well.

**Problem 7.**

(a) Converges; use  $\ln(n) \leq n^{1/4}$  for large  $n$  and compare the terms to  $n^{-5/4}$ .

(b) Diverges; as  $\sqrt[n]{n} \rightarrow 1$ , by the  $n$ -th term test.

**Problem 8.**

$$(a) |\overrightarrow{AB}| = \sqrt{3}, \quad |\overrightarrow{Ac}| = 2$$

$$(b) \cos(\alpha) = -1/\sqrt{3}$$

$$(c) d = |\overrightarrow{AB}| \sin(\alpha) = \sqrt{2}$$

using the fact that:  $\sin(\alpha) = \sqrt{1 - \cos^2(\alpha)} = \sqrt{2/3}$ .