MATH 2260 - PRACTICE EXAM III.

There are more problems and more parts (with some challenging ones) than there will be on the exam, to give you more practice. The problems are representative however.

Problem 1. Decide whether each of the following series converges. If a given series converges, compute its sum.

a) $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+1}}$

b)* $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

c) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

d)* $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$

Problem 2. Use the integral test to decide which of the following series converge. In each case write down the integral and compare to the sum of the series. (Its a good practice to try to use other tests as well to determine to convergence of the series below.)

a) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

b) $\sum_{n=1}^{\infty} ne^{-n^2}$

c) Use the integral test to show that: $\ln(N+1) \leq \sum_{n=1}^{N} \frac{1}{n} \leq ln(N) + 1$

d) $\sum_{n=2}^{\infty} \frac{1}{n\ \ln(n)^2}$

Problem 3. Use the comparison or limit comparison test to decide which of the following series converge.

a) $\sum_{n=1}^{\infty} \frac{n\ \sin^2(n)}{n^3+1}$

b) $\sum_{n=1}^{\infty} \frac{\ln(n)^3}{n^2}$

c) $\sum_{n=1}^{\infty} \frac{2^{2n}}{5^n+n+1}$

d) $\sum_{n=1}^{\infty} \sin(1/n)$

Hint: Use the basic trig. limit: $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.

Problem 4. Use the ratio test or the root test to determine the convergence of the following series.

a) $\sum_{n=1}^{\infty} n^2 3^{n+10} 4^{-n}$

b) $\sum_{n=1}^{\infty} \frac{(n+1)^1 010^n}{n!}$

c) $\sum_{n=1}^{\infty} \frac{(n^2+10)^n}{(2n-1)^{2n}}$

d $\sum_{n=1}^{\infty} n!\, 2^{-n^2}$

Problem 5. Decide whether the following series converge absolutely, converge conditionally or diverge.

a) $\sum_{n=1}^{\infty} (n+1) \sin\left(\frac{1}{n^3}\right)$

b) $\sum_{n=1}^{\infty} (-1)^n n^{-1/2}$

c) $\sum_{n=1}^{\infty} (-2)^n n^{-10}$

Problem 6. Find the values of x for which the following power series converge.

a) $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}+1}$

b) $\sum_{n=0}^{\infty} 10^{-n-1}(x-1)^n$

c) $\sum_{n=1}^{\infty} n!\, 2^{-n^2}\, x^n$

Problem 7. Expand the following functions as power series around $a = 0$, that is in the form: $f(x) = \sum_{n=0}^{\infty} a_n x^n$.

a) $f(x) = \frac{1}{1+x^3}$

b) $f(x) = e^{-2x}$

c)* $f(x) = xe^{-x^2}$

d)* $f(x) = \ln(1+x)$

Hint: First find the power series expansion of e^{-x^2} by an appropriate substitution and the multiply each term with x. Use the power series expansion of the anti-derivative of a function, or alternatively the Taylor series formula, for part d).