

Math 8200, Spring 2010

Homework 1: due Tuesday 19 January at the beginning of class Homotopy, homotopy equivalence and deformation

Reading

- Chapter 0: Read pages 1-4: you should know most of this (at the moment you can skip the ‘mapping cylinder’, but this is an important idea). In particular, try to understand the ‘house with two rooms’ example and why it is contractible. Know the meaning of ‘wedge sum’ (page 10, but ignore references to cell complexes for now). Look at Examples 0.7, 0.8, 0.9.

Problems

1. (10 points) Let X and Y be topological spaces. Let $\text{Map}(X, Y)$ denote the set of all continuous maps from X to Y . Show that ‘being homotopic’ is an equivalence relation on the set $\text{Map}(X, Y)$.
2. (10 points) Let Top denote the class of all topological spaces. Show that ‘being homotopy equivalent’ is an equivalence relation on the class Top .
3. (5 points) Suppose that the topological spaces X and Y are homeomorphic. Prove that X and Y are homotopy equivalent.
4. (10 points) Let X be any topological space, and $f, g : X \rightarrow \mathbb{R}^n$ any continuous maps. Prove that f and g are homotopic. (Hint: Example 1.1 on page 25.)
5. (10 points) Let X be a topological space and let $x \in X$. Prove that X is contractible if and only if the identity on X is homotopic to the constant map $c_x : X \rightarrow X$ given by $c_x(y) = x$ for all $y \in X$.
6. (5 points) Show that the space $\mathbb{R}^2 - \{(0, 0)\}$ is homotopy equivalent to S^1 . For this question only, I do want you to give **explicit formulas** for any homotopies or deformation retractions that you use.
7. (5 points) Show that the space $\mathbb{R}^2 - \{(-1, 0), (1, 0)\}$ is homotopy equivalent to $S^1 \vee S^1$.

8. (5 points) Show that the space $\mathbb{R}^3 - \{(-1, 0, 0), (1, 0, 0)\}$ is homotopy equivalent to $S^2 \vee S^2$.
9. (5 points) Exercise 5 on page 18.
10. (5 points) Exercise 6(a) on page 18.
11. (5 points) Exercise 20 on page 19.

The following problems are harder and some require you to read more of the textbook. You may substitute as many of these for whichever of the above questions you like although your total score cannot be above 75.

12. (10 points) Exercise 6(b,c) on page 18.
13. (10 points) Exercise 13 on page 19.
14. (10 points) Exercise 27 on page 20.