Math 8200: Homework 10: due Friday 2 April

Please put your homework directly into Jim Stankewicz's mailbox by 5pm on Friday.

Reading

- We have diverged a little bit from the approach the textbook uses. It covers the Mayer-Vietoris sequence later in section 2.2 and uses a slightly different method (based on reduced homology) to calculate the singular homology of spheres and to prove that singular and simplicial homology are isomorphic. You should look at Proposition 2.22 which we haven't covered in class, and then go back and look at 2.13 - 2.15. The section titled 'Naturality' on page 127 is important because it gives us the maps between long exact sequences that we have used in various places. Then read the book's proof of 2.27. The main difference between this and our proof in class is the method for calculating $H_n(X_k, X_{k-1})$: it uses 2.22, where I did it directly from excision.

  In section 2.2 (page 149) the book gives a different derivation of the Mayer-Vietoris sequence than I did in class. The main thing to concentrate on at the moment is working through examples so look at 2.46 and 2.47.

Problems

Each problem is worth 5 points.

1. Let CX denote the *cone* on the topological space, defined at the top of page 9. Prove that for any space X, CX is contractible.

2. Write out a clear proof of the following statement, with as many details as you can:

   - Let X be a infinite-dimensional Δ-complex and let X^k denote the k-skeleton of X. For each $\sigma \in C_n(X)$, there is an integer k such that σ is in the subgroup $C_n(X^k)$.

3. Use the Mayer-Vietoris sequence to calculate the singular homology groups of $\mathbb{R}P^2$. (Hint: view $\mathbb{R}P^2$ as a copy of D^2 with antipodal points on the boundary identified. Recall that the boundary is then a subspace A of $\mathbb{R}P^2$ that is homeomorphic to $\mathbb{R}P^1$. Take one of the open sets to be a neighbourhood U of A, such that U deformation retracts onto A.)

4. Use the Mayer-Vietoris sequence to calculate the singular homology groups of the orientable genus 2 surface $T\#T$ where $T = S^1 \times S^1$.

5. #19 on page 132 (this was one of the harder problems on the previous homework but with the Mayer-Vietoris sequence this is now fairly easy)

6. #31 on page 133

7. #28 on page 157 (part (a) only)

8. #31 on page 158 (use the Mayer-Vietoris sequence for reduced homology in which all the homology groups are replaced with reduced homology groups)

9. #32 on page 158 (use the Mayer-Vietoris sequence for reduced homology, look on page 8 for the definition of the suspension of a space)

10. #38 on page 159 (we essentially used this result to deduce the existence of the Mayer-Vietoris sequence from excision and the long exact sequences for relative homology)

Harder Problems

You may substitute any of these problems for those above (though your total score cannot be more than 50 points). Each problem is worth 5 points.

11. #23 on page 133

12. #35 on page 158

13. The sphere S^n has the property that its only nonzero reduced homology group is $\tilde{H}_n(S^n) = \mathbb{Z}$. Find a space X whose only nonzero reduced homology group is $\tilde{H}_n(X) = \mathbb{Z}/2$.