Math 8200: Homework 11: due Friday 9 April

Please put your homework directly into Jim Stankewicz's mailbox by 5pm on Friday.

Reading

- We have now moved onto section 2.2. Read the section on 'Degree' starting on page 134. In particular note property (g) on fixed points which we did not talk about in class. We did Theorem 2.28, but look also at Proposition 2.29 and 2.31, 2.32 and 2.33.

  We have also started looking at CW-complexes. For the definition of these, go back to chapter 0 and page 5. Of particular importance here are understanding all the examples on page 6. Also look at the definition of subcomplex on page 7. The section 'Operations on Spaces' is also important. For example, you will need to understand the CW-complex structure on $X \times Y$, when X and Y are CW-complexes, in order to do one of the homework problems. You will also need the general definition of Euler characteristic for a finite CW complex given on page 146.

Problems

All the regular problems are from the textbook this week. Each problem is worth 5 points.

- Page 155: #1, 2, 3, 4, 7, 8
- Page 19: #14, 19
- Page 157: #20, 21

Harder Problems

For each CW complex X, it is possible to define abelian groups $K_n(X)$, one for each $n \in \mathbb{Z}$ (not just positive n), that satisfy the following properties:

- a continuous map $f : X \to Y$ determines a homomorphism $f_* : K_n(X) \to K_n(Y)$ such that $(g \circ f)_* = g_* \circ f_*$ and $(1_X)_* = 1_{K_n(X)}$
- if $f, g : X \to Y$ are homotopic, then $f_* = g_*$
- there is a Mayer-Vietoris sequence, i.e. whenever $X = U \cup V$ for open sets U, V, there is a long exact sequence

$$\ldots K_n(U \cap V) \to K_n(U) \oplus K_n(V) \to K_n(X) \xrightarrow{d_n} K_{n-1}(U \cap V) \to \ldots$$

- if $\emptyset$ denotes the empty space, then $K_n(\emptyset) = 0$ for all n
- if X is the one-point space, then

$$K_n(X) = \begin{cases} \mathbb{Z} & \text{if } n \text{ is even;} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

Notice that the ordinary homology groups satisfy the same properties except for the last. Also notice that $K_n(X)$ can be nonzero for negative n. For example $K_{-2}(\bullet) = \mathbb{Z}$ where $\bullet$ is the one-point space.
You may substitute each of the following for one of the regular problems. Each is worth 5 points.

1. Calculate $K_n(S^r)$.
2. Calculate $K_n(\mathbb{R}P^r)$.

The groups $K_n(X)$ are the *complex K-homology groups of* X. They are defined by looking at vector bundles on X. (A vector bundle is like a covering space except that each inverse image is a vector space instead of a discrete set.)