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★Spline functions on triangulations.

Encyclopedia of Mathematics and its Applications, 110.
Cambridge University Press, *Cambridge*, 2007. *xvi*+592 *pp*. $130.00.
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By “spline function” the authors mean a smooth (usually once or twice differentiable) piecewise polynomial function defined on a partition of a two- or three-dimensional domain, usually into triangles or tetrahedra.

Univariate splines (smooth piecewise polynomial functions of one variable, defined on a partition of an interval) were developed mostly in the 1960s and ’70s. They are used ubiquitously in a wide range of application areas and now form a standard topic in numerical analysis textbooks. The bulk of the work on multivariate splines started in the late ’70s, reached its heyday in the ’80s and ’90s, and is ongoing. Multivariate splines have applications in areas such as scattered data interpolation, function approximation, geometric design, surface representation and processing, and the numerical solution of partial differential equations.

The monograph under review here is the definitive account of the state of the subject through the year 2007. It is authoritative, comprehensive, accurate, thoughtful, and extremely well and carefully written. In many ways it can be considered the companion of L. L. Schumaker’s well-known and classic monograph [*Spline functions: basic theory*, Wiley, New York, 1981; MR0606200 (82j:41001)], which covers univariate and tensor product splines. Recently, that book appeared in its third edition [Cambridge Univ. Press, Cambridge, 2007; MR2348176].

There are 18 chapters, and about 360 references are quoted in the text. (There is also a reference in the preface to a much larger online bibliography.) Each chapter concludes with Remarks and detailed Historical Notes. The book can be used as a textbook for a graduate class, but it does not contain formal exercises.

Specific covered topics include: triangulations and tetrahedral partitions, bi- and trivariate polynomials, Bernstein-Bézier form and techniques, approximation power of splines and polynomials, interpolation by splines and polynomials, dimensions of spline spaces, (nodal, local, stable) bases of spline spaces, minimal determining sets, macro elements, finite elements, box splines, splines on the sphere.

If you need to know anything about multivariate splines this book will be your first and surest source of information for years to come.

Reviewed by *Peter Alfeld*