Bivariate Splines for Hurricane Path Forecasting

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1 Introduction

Every year, hurricanes cause a lot of damage, especially, when they hit cities along the coast line. A notorious example is Hurricane Katrina in 2005 which hit New Orleans and damaged the city significantly. Human intervention cannot change the path of a hurricane. Hence the best possible way to reduce the damage is to predict the path of the hurricane and avoid direct contact. The formation, movement and strength of hurricanes is not understood very well. Although meteorologists have several methods to predict the path of hurricanes, current methods are still not able to predict the paths with enough lead time, accuracy and confidence to evacuate and prepare cities. For example, Hurricane Rita in 2005 was wrongly predicted to hit Houston which caused tens of thousands residents to evacuate for nothing. Little is known about the physical, chemical and thermodynamic explanations of the speed and direction of the movement of a hurricane. Hurricanes seem to move randomly. Nevertheless, people are determined to understand the phenomenon of hurricane. Satellites are constantly observing hurricanes from space. Airplanes track hurricanes from the sky. Buoys are placed in the oceans to collect hourly barometric information. In particular, after Hurricane Katrina, more buoys were placed in the Gulf of Mexico. Several technologies have been initiated and developed to decipher the data and information to predict the path of a hurricane. See Figures 1 and 2 for buoy locations in the Gulf of Mexico below.

We approach this prediction problem by using bivariate spline functions. Although univariate splines have been applied for predictions (cf. (2), (4), (5) and (6)), bivariate splines have never been used for hurricane predictions. Bivariate spline functions have the advantage of fitting scattered data conveniently. We find a se-

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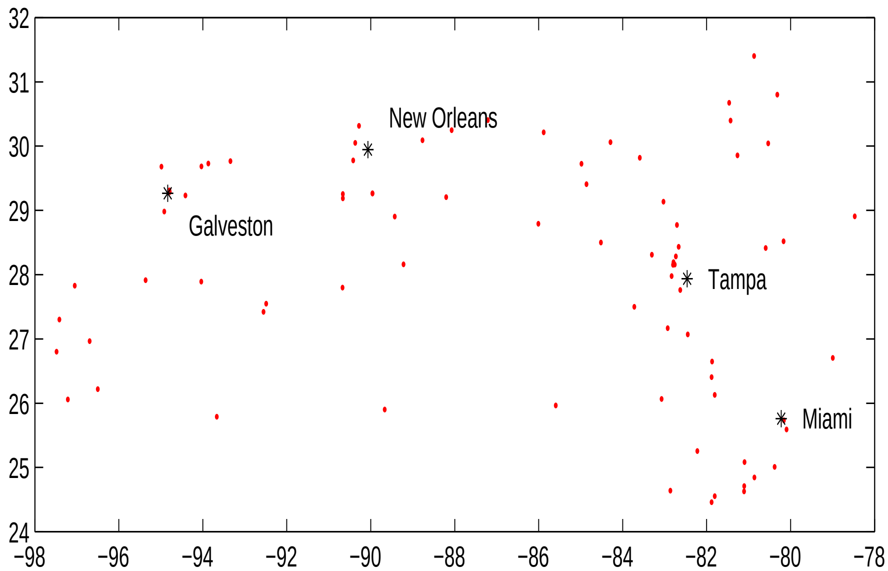

Fig. 1 Buoy Locations for the 2005 Hurricane Season

quence of spline functions by fitting the barometric values from the buoys every hour. We refer the interested reader to (16) for the theory of multivariate spline functions and (1) for the computational algorithms to find data fitting splines. The eye of hurricane causes a vacuum where the barometric value will be lowest. The lowest values of these spline functions over a period of time track of the movement of a hurricane. To predict the path of a hurricane, we have to use more mathematics. Many prediction theories in statistics are available (cf. (7), (10), (17)). We find that the so-called "Brute Force Method" developed by Guillas and Lai recently in (13) is particularly useful. They applied the method to study the prediction of ozone concentration in Atlanta. Many numerical experiments have been performed which demonstrate the effectiveness of their method (See [Ettinger, 2008(8)] for the more numerical experiments in addition to those presented in (13)). There are two advantages: One is consistency. That is, various lengths of learning periods give almost the same prediction. The other advantage is the accuracy. The method gives good predictions for more than 9 days during two consecutive weeks in an experiment.

However, we cannot use the method directly as there will be no buoys measuring the barometric data along the path of a hurricane. The method has to adapt to predicting the path of a hurricane. In Section 2, we give a preliminary for bivariate splines and scattered data fitting. Then in Section 3, we explain Guillas and Lai's Brute Force Method. Section 4, outlines our numerical experiments. In Section 5, we test our method by using the buoy data to predict the path of hurricanes Katrina in 2005 and Ike in 2008. Note that the Katrina knocked out almost all buoys when it hit New Orleans at 9:00 a.m. on August 29, 2005. We have the barometric data every hours for two days before 9:00 a.m. Also in 2005, we only have 80 buoys in the

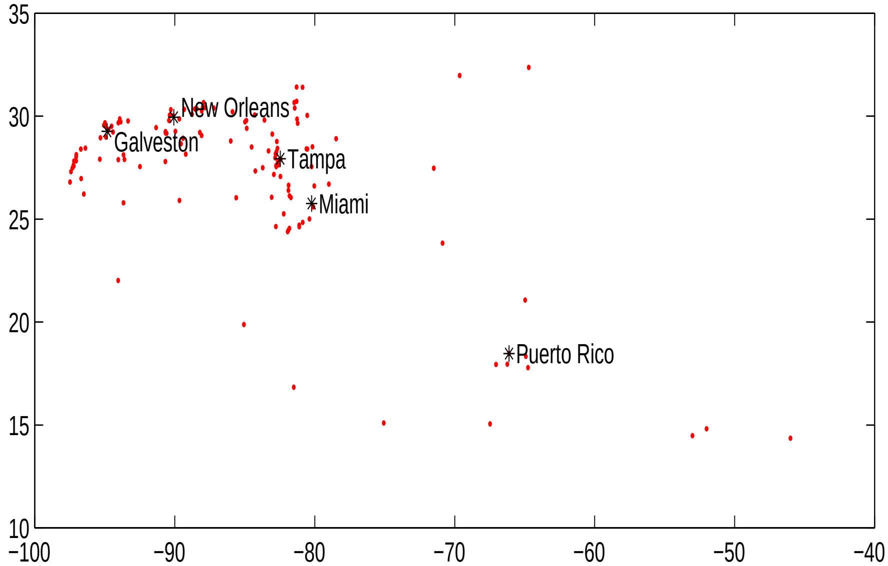

Fig. 2 Buoy Locations for the 2008 Hurricane Season

Gulf of Mexico. Although these information may not be enough, we are still able to predict the Katrina within 43 nautical miles based on 6 hour lead time and 20 nautical miles based on 7 hour lead time. After Katrina, more buoys were placed in the Gulf of Mexico and barometric data were collected every hour. We have tested our prediction method for Hurricane Ike in 2008. Numerical results show that we are able to predict the Ike with more stability. These numerical results will be reported at the end of the paper.

2 A Preliminary for Bivariate Spline Theory and Scattered Data Fitting

In this section, we review some basics of bivariate splines and the necessary spline theory we need for our application to functional linear models. Most of the spline results presented in this section can be found in [Lai and Schumaker'07, (16)]. Let $\mathscr{D}$ be a polygonal domain in $\mathbb{R}^2$ and $\triangle$ a triangulation of $\mathscr{D}$. That is, $\triangle$ is a finite collection of triangles $T \subset \mathscr{D}$ such that $\cup_{T\in\triangle} T = \mathscr{D}$ and the intersection of any two triangles is either the empty set, a common edge, or a common vertex. For each $T \in \triangle$, let $|T|$ denote the length of the longest edge of T, and let ρ_T be the radius of the inscribed circle of T. The longest edge length in the triangulation $\triangle$ is denoted by $|\triangle|$ and is referred to as the size of the triangulation. For any triangulation $\triangle$ we define its shape parameter by

$$\kappa_{\triangle} := \frac{|\triangle|}{\rho_{\triangle}}, \tag{1}$$

where $\rho_{\triangle}$ is the minimum of the radii of the in-circles of the triangles of $\triangle$. The shape parameter for a single triangle, κ_T, satisfies

$$\kappa_T := \frac{|T|}{\rho_T} \leq \frac{2}{\tan(\theta_T/2)} \leq \frac{2}{\sin(\theta_T/2)}, \tag{2}$$

where θ_T is the smallest angle in the triangle T. The shape of a given triangulation affects how well we can approximate a function over the triangulation. Hence we have the following definition of a β-quasi-uniform triangulation.

Definition 1 (β-Quasi-Uniform Triangulation). Let $0 < \beta < \infty$. A triangulation $\triangle$ is a β-*quasi-uniform* triangulation provided that

$$\frac{|\triangle|}{\rho_{\triangle}} \leq \beta.$$

Once we have a triangulation, we define the spline space of degree d and smoothness r over that triangulation as follows:

Definition 2 (Spline Space). Let $\triangle$ be a given triangulation of a domain $\mathscr{D}$. Then we define the spline space of smoothness r and degree d over $\triangle$ by,

$$S_d^r(\triangle) = \{s \in C^r(\mathscr{D}) \mid s|_T \in \mathscr{P}_d, \ \forall\, T \in \triangle\},$$

where $\mathscr{P}_d$ is the space of polynomials of degree at most d.

When $d \geq 3r+2$ the spline space $S_d^r(\triangle)$ possesses an optimal approximation order which is achieved by the use of a quasi-interpolation operator. To define the quasi-interpolation operator we need linear functionals $\{\lambda_{ijk,T}\}_{i+j+k=d}, T \in \triangle$ which are based on values of f at the set of domain points over triangles in $\triangle$, that is

$$\lambda_{ijk,T}(f) = \sum_{|\nu|=d} a_\nu^{ijk} f(\xi_\nu^T). \tag{3}$$

The quasi-interpolation operator of f is defined by

$$Qf := \sum_{T\in\triangle} \sum_{i+j+k=d} \lambda_{ijk,T}(f) B_{ijk}^T. \tag{4}$$

Now, we are ready for the theorem on optimal approximation order [Lai and Schumaker'98, (15)].

Theorem 1 (Optimal Approximation Order). *Assume $d \geq 3r+2$ and let $\triangle$ be a triangulation of $\mathscr{D}$. Then there exists a quasi-interpolatory operator $Qf \in S_d^r(\triangle)$ mapping $f \in L_1(\mathscr{D})$ into $S_d^r(\triangle)$ such that Qf achieves the optimal approximation order: if $f \in W_p^{m+1}(\mathscr{D})$,*

$$\|D_1^\alpha D_2^\beta (Qf - f)\|_{L_p(\mathscr{D})} \leq C|\triangle|^{m+1-\alpha-\beta} |f|_{m+1,p,\mathscr{D}} \tag{5}$$

for all $\alpha+\beta \le m+1$ with $0 \le m \le d$, where D_1 and D_2 denote the derivatives with respect to the first and second variables, $\|f\|_{L_p(\mathscr{D})}$ denotes the usual L_p norm of f over $\mathscr{D}$, $|f|_{m,p,\mathscr{D}}$ denotes the L_p norm of the m^{th} derivatives of f over $\mathscr{D}$, and $W_p^{m+1}(\mathscr{D})$ stands for the usual Sobolev space over $\mathscr{D}$. The constant C depends only on the degree d and the smallest angle $\theta_\triangle$ and may be dependent on the Lipschitz condition on the boundary of $\mathscr{D}$.

Next we explain a scattered data fitting method based on bivariate splines. We explain the so-called *Penalized Least Squares Spline Method*. Many other methods were explained in [Lai,2008,(14)]. Suppose that we are given a set of scattered data: $\{(x_i, y_i, f_i), i = 1, \cdots, n\}$ with data locations $\{(x_i, y_i), i = 1, 2, \cdots n\} \subset \mathscr{D}$. For each $s \in S_d^r(\triangle)$, let $\mathscr{L}(s-f) = \frac{1}{n}\sum_{i=1}^{n} |s(x_i, y_i) - f_i|^2$ be the discrete L^2 semi-norm and

$$E_m(f) = \frac{1}{A_{\mathscr{D}}}\int_{\Omega} \sum_{i+j=m} \left(\binom{m}{i} (D_1^i D_2^j f)^2 \right) dx_1 dx_2$$

be the energy functional, where $m \ge 1$ is a fixed integer and $A_{\mathscr{D}}$ is the area of $\mathscr{D}$. Typically, one uses $m = 2$. One solves the minimization problem

$$\min_{s \in S_d^r(\triangle)} \mathscr{L}(s-f) + \lambda E_m(s) \tag{6}$$

for a fixed integer $m = 2$ and $\lambda > 0$. The solution s_f is called the penalized least squares spline of the given data. We use this method to compute a bivariate spline fit to the hurricane data to be discussed in the next section. A computational algorithm for penalized least squares splines was explained in [Awanou, Lai, and Wenston'06(1)]. For the approximation properties of penalized least squares splines, we refer to [Golitschek and Schumaker'02(11), (12)].

3 Brute Force Method

The general idea of the "Brute Force Method" from [Guillas and Lai'10, (13)] starts by approximating a bounded and continuous functional f. The main point is that a future value (to be predicted) is assumed to be a linear functional of the current function over a bounded domain $\mathscr{D}$ with some nonlinear noise. That is,

$$y = f(X) + \varepsilon,$$

where y is a future value, f is a continuous linear functional of current random surface X and ε is nonlinear noise. The Riesz representation theorem says, the functional can be written as $f(X) = \langle g, X\rangle$ for some unknown function $g \in H$. Hence one could approximate f by solving the minimization problem

$$\alpha = \arg\min_{\beta \in H} \mathscr{E}\left[(f(X) - \varepsilon - \langle \beta, X \rangle)^2\right], \tag{7}$$

where H stands for a Hilbert space, say $L^2(\mathscr{D})$. Although, the above minimization problem is impossible to solve because the solution lies in the infinite dimensional Hilbert space H. We approximate the solution by choosing a finite dimensional spline space $S_d^r(\triangle)$ of smoothness r and degree $d \geq 3r+2$ which will be dense in H as $|\triangle| \to 0$ by the optimal approximation order of splines, see Theorem 1. This reduces the original problem (7) to the spline estimate

$$S_\alpha = \arg\min_{\beta \in S_d^r(\triangle)} \mathscr{E}\left[(f(X) - \varepsilon - \langle \beta, X \rangle)^2\right]. \tag{8}$$

The following theorem (cf. (13)) states the existence and uniqueness of a solution to the (8) over $S_d^r(\triangle)$.

Theorem 2. *Suppose that only the zero spline in $S_d^r(\triangle)$ is orthogonal to the collection of random surfaces $\mathscr{X} = \{X(s),\ s \in \mathscr{D}\} \subset H$. Then the minimization problem (8) has a unique solution in $S_d^r(\triangle)$.*

We can extend the result above to an empirical estimate for a set of observed surfaces $\{X_1, \ldots, X_n\}$ by taking

$$\widetilde{S_{\alpha,n}} = \arg\min_{\beta \in S_d^r(\triangle)} \frac{1}{n}\sum_{i=1}^{n}(f(X_i) - \varepsilon_i - \langle \beta, X_i \rangle)^2. \tag{9}$$

In (13), we can find the following result that the empirical estimate inherits similar properties for existence and uniqueness.

Theorem 3. *Suppose that only the zero spline function in the spline space $S_d^r(\triangle)$ is perpendicular to the subspace span$\{X_1, \cdots, X_n\}$ except on an event whose probability p_n goes to zero as $n \to +\infty$. Then, with probability $1 - p_n$, there exists a unique $\widetilde{S_{\alpha,n}} \in S_d^r(\triangle)$ minimizing (9).*

So far we have only been discussing cases where the given observations are complete random surfaces. However in practice, we are not able to observe an entire random surface. Instead, we observe a random surface X over design points $s_k, k = 1, \ldots, N$ in $\mathscr{D}$. We create a surface to represent X from the given data by using a penalized least squares spline. That is, we create a spline S_X by finding the penalized least square fit of X as we explained in the previous section.

We thus modify the original problem (7) using S_X instead of X:

$$\alpha_D = \arg\min_{\beta \in H} \mathscr{E}\left[(f(X) - \varepsilon - \langle \beta, S_X \rangle)^2\right]. \tag{10}$$

Again we have the same issue, that the Hilbert space H is infinite and we may not be able to find the minimum. However, we can find an approximate solution in the finite spline space $S_d^r(\triangle)$. Thus we obtain a spline estimate based on the approximated random surfaces:

$$S_{\alpha_D} = \arg \min_{\beta \in S^r_d(\triangle)} \mathscr{E}\left[(f(X) - \varepsilon - \langle \beta, S_X \rangle)^2 \right]. \tag{11}$$

In the next theorem we see how well S_{α_D} approximates α_D in terms of $|\triangle|$, the size of triangulation. See (13) for a proof.

Theorem 4. *Suppose that $\mathscr{E}(\|X\|^2) < \infty$ and suppose $\alpha \in C^r(\mathscr{D})$ for $r \geq 0$. Then the solution S_{α_D} from the minimization problem (11) approximates α_S in the following sense:*

$$\mathscr{E}((\langle \alpha_D - S_{\alpha_D}, S_X \rangle)^2) \leq C|\triangle|^{2r} \tag{12}$$

for a constant C dependent on $\mathscr{E}(\|X\|^2)$, where $|\triangle|$ is the maximal length of the edges of $\triangle$.

From the above spline estimate we compute an empirical estimate for real data based the approximated random surfaces:

$$\widehat{S_{\alpha,n}} = \arg \min_{\beta \in S^r_d(\triangle)} \frac{1}{n} \sum_{i=1}^{n} (f(X_i) - \varepsilon_i - \langle \beta, S_{X_i} \rangle)^2. \tag{13}$$

Once we have $\widehat{S_{\alpha,n}}$, we can use it to predict the future value by computing the inner product $\langle S_{X_n}, \widehat{S_{\alpha,n}} \rangle$. This is the so-called the Brute-Force Method. This method works very well for ozone concentration prediction at Atlanta as demonstrated in (13), (9), and (8). However to predict the path of a hurricane, we have to extend this approach since may not be any observations along the path of the hurricane. The extension will be discussed in the next section.

4 Numerical Method For Hurricane Path Prediction

To predict the path of a hurricane in the Gulf of Mexico, we study the barometric pressures over a polygonal domain which approximates a region of interest within the Gulf of Mexico. Note that the eye of a hurricane is where the barometric pressure is lowest. The path of a hurricane over the course of several days is the trace of the eye of the hurricane over that period of the time. Mainly, we predict the future barometric pressure based on the current barometric pressure surface. That is, we assume that both the current explanatory and future response variables are random surfaces. The future barometric pressure surface can be approximated by penalized least squares spline fitting to the future barometric pressure values over the scattered locations of buoys. All we need to do is to find these future barometric pressure values. They can be predicted by using the Brute Force Method at each location. This explains our method for the prediction of the path of a hurricane. Let us rephrase our approach by starting from the beginning of our computation. At each buoy location, we use Guillas-Lai's Brute Force Method to predict the future value of barometric pressure at that location. After we obtain the future values at all buoy locations, we fit a bivariate spline through these predicted values over the buoy locations by using penalized least squares spline method. This fitted spline is the future (or predicted) barometric pressure surface. Then we compute the location of the minimum of the future barometric pressure surface to be the prediction of the eye of the hurricane.

Our main assumption is that the barometric pressure for a specific time and location is a linear functional of previous barometric pressure surfaces from certain hours earlier. For each prediction, we fix the length of the prediction P and the number L of learning periods. For example to make a six hour prediction, $P=6$, for August 29, 2005 at 9:00 a.m., we assume that barometric pressure on August 29 at 9:00 a.m. at a particular buoy in the Gulf of Mexico is a linear functional of the barometric pressure surface over the Gulf of Mexico six hours earlier, August 29 at 3:00 a.m.. For a six hour prediction, we approximate the linear functional by learning over several six hour periods. For each learning period, we let Y_i be the hourly barometric pressure at a particular buoy in the Gulf of Mexico and X_i is the barometric surface over the Gulf of Mexico from six hours earlier. Then we find the spline that minimizes (13). To do this, we write $\widehat{S_{\alpha_D,n}} = \sum_{i=1}^m \hat{c}_{n,i}\phi_i$, where $\phi_i, i=1,\cdots,m$ are locally supported basis functions for $S^r_d(\triangle)$ and rewrite the minimization problem as the linear system

$$Ac = b, \tag{14}$$

where

$$A = \frac{1}{n}\sum_{l=1}^{n}\langle \phi_i, S_{X_l}\rangle\langle \phi_j, S_{X_l}\rangle \quad \text{and} \quad b = \frac{1}{n}\sum_{l=1}^{n} Y_i\langle \phi_j, S_{X_l}\rangle.$$

The coefficient vector of $\widehat{S_{\alpha_D,n}}$ is the vector $\mathbf{c}_n = (c_{n,i}, i=1,\ldots,m)$ that satisfies $Ac = b$. Then for each buoy, we use the buoy specific spline $\widehat{S_{\alpha_D,n}}$ to make a prediction for that specific buoy by taking the inner product with the last known Y_i at that location,

$$\langle Y_i, \widehat{S_{\alpha_D,n}}\rangle \approx Y_{i+P}. \tag{15}$$

Finally, we create a prediction surface by fitting a spline surface to all the predicted values.

Our domain is a region of interest in the Gulf of Mexico, and our design points are the ocean buoys where the barometric pressure values are collected. For our numerical experiments, we have scaled the domain into $[0,1]\times[0,1]$, see Figure 3 and Figure 4. In 2005, we only have 80 buoys in our domain. For 2008, there are 140 buoys in our domain and the domain covers a larger area of the Gulf of Mexico. For each buoy, we have hourly barometric pressure readings for the 2005 and 2008 Hurricane seasons. Some data values are missing as hurricanes tend to knock out the buoys. The missing values do not affect our computation because the missing values are approximated by the input surface.

Below is a brief outline of the numerical experiments. To get a better idea of the process, we will set up the outline for a six hour prediction for August 29 at 9:00 a.m. over one learning period.

Step 1) For each hour from August 28 at 4:00 p.m. through 9:00 p.m., we create the

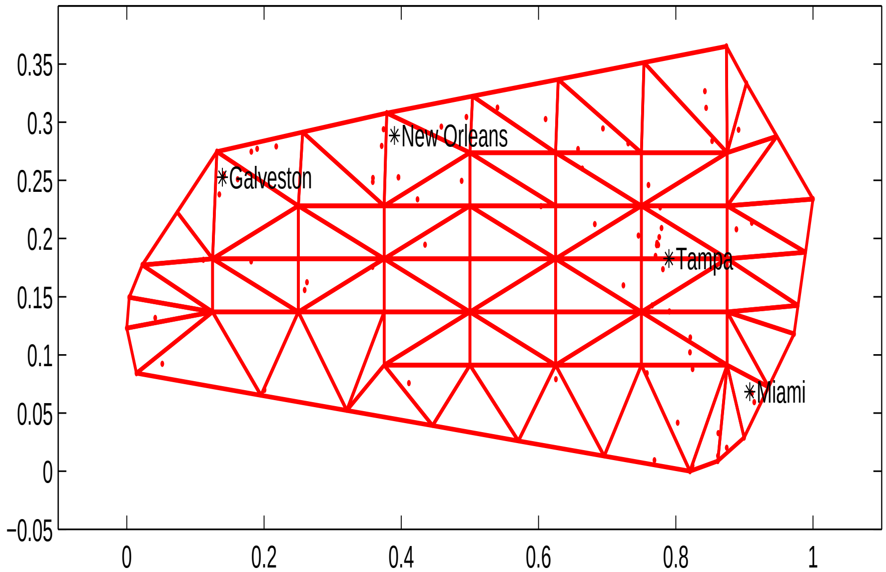

Fig. 3 Triangulation for 2005 Season with Buoy Locations

approximation surface S_{X_i} by fitting a penalized least squares spline with penalty $\lambda = 10^{-5}$ over the spline space with degree 5 and smoothness 1 based on the avail-

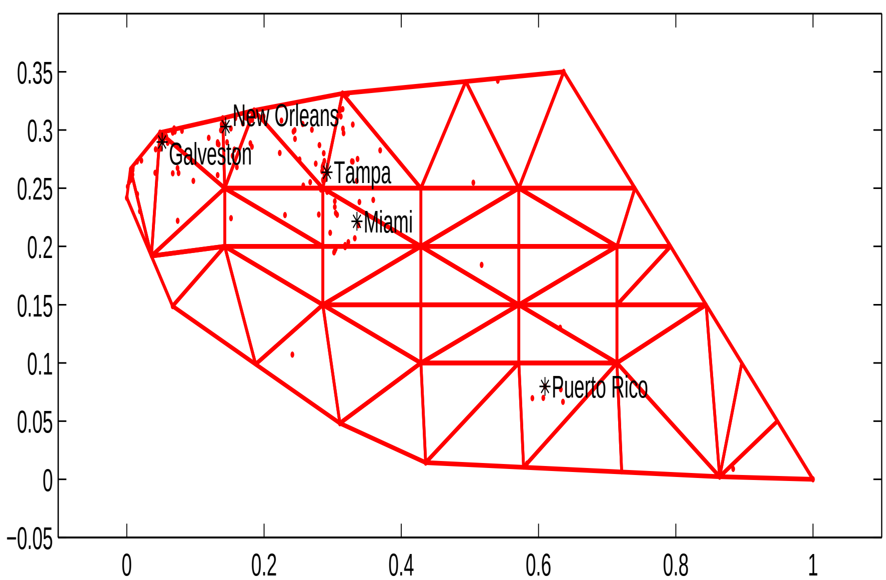

Fig. 4 Triangulation for 2008 Season with Buoy Locations

able buoy data.
Step 2) Then for each buoy, we collect Y_i the barometric pressure six hours ahead of X_i. We set up and solve the linear system (14) described above with $n = 6$ for $L = 1$ and $n = 12$ for $L = 2$ and $n = 18$ for $L = 3$. This step yields a $\widehat{S_{\alpha_D,n}}$ for each buoy in the domain for different L.

Step 3) To make a $P = 6$ hour prediction, we first compute the least squares spline S_Y fitting to the Y_i's at 3:00 a.m. on August 29 and then by taking the inner product of the last known barometric pressure surface S_Y with each prediction spline $\widehat{S_{\alpha_D,n}}$ from Step 2 to obtain the predicted value at each buoy for 9:00 a.m., August 29. That is, we take $\langle S_Y, \widehat{S_{\alpha_D,n}} \rangle$ to give us a six hour prediction for each buoy in the domain. For our specific example, the last known barometric pressure values Y_i are on 3:00 a.m. on August 29.
Step 4) Finally, we fit a spline surface to all of the buoy predictions for August 29 at 9:00 a.m. to create an approximate barometric pressure surface at 9:00 a.m. on August 29. The minimum of this surface is the predicted location of the eye of the hurricane.

We use the same steps for other prediction lengths. Numerical results are collected in the next section.

5 Numerical Results

In this section, we show two numerical examples. The first is hurricane Katrina from 2005. The second is Hurricane Ike from 2008.

Example 1 (Katrina). For Hurricane Katrina, we first list the contour of barometric pressure data at landfall 9:00 a.m. on August 29, 2005. The landfall surface, August 29, 2005 at 9:00 a.m., is the surface we are trying to predict. Then we list the last known surface for the six, seven, eight, nine and ten hour predictions. For each the 3:00 a.m., 4:00 a.m.,..., 1:00 a.m. August 29,2005 and 12:00 p.m. and 11:00 p.m., August 28, 2005 for the exact measurements based on penalized least squares splines to the barometric pressure data from 80 buoys over the Gulf of Mexico are shown in Figure 7.

In Figure 8, we list the predicted contour of barometers on 9:00 a.m., August 29, 2005 based on our prediction approach using 6 hour learning, 7 hour learning, 8 hour learning, 9 hour learning, 10 hour learning as well as 11 hour learning.

To see how accurate we can predict, we list the accuracy in Nautical Miles in Figure 5. Note that 1 Nm (Nautical Mile) is equal to 1.15077945 mile. Each row contains a prediction length in hours from $1, 2, \cdots, 12$ hours in advance. The columns are the number of learning periods, $L = 1, 2, 3$.

	1 Period	2 Periods	3 Periods
1	19.16	15.18	15.18
2	15.18	28.92	33.73
3	25.22	47.04	61.99
4	37.02	36.50	32.69
5	56.02	46.23	28.92
6	66.00	43.05	59.02
7	35.65	20.28	228.23
8	222.20	228.23	222.20
9	59.83	229.81	222.20
10	228.23	222.43	222.20
11	135.01	228.23	222.20
12	228.23	222.20	222.20

Fig. 5 Errors in Nautical Miles for Hurricane Katrina

From the Table above, we can see that when we use 6 hour lead time and 7 hour lead time with 2 learning periods, we can predict the location of the eye of Katrina within 43Nm and 20Nm of New Orleans. These are very good predictions!

Example 2 (Ike). In this example, we study the prediction of Hurricane Ike. The number of buoys is 140 which is many more than the number of buoys for Hurricane Katrina. In Figure 9, we show the contours of the barometric pressure for the exact measurements based on penalized least squares splines to the barometric pressure data from 140 buoys over the Gulf of Mexico at 9:00 a.m., on September 24, 2008, as well as 3:00 a.m., 2:00 a.m., and 1:00 a.m. on September 24, 2008 and 11:00 p.m. and 12:00 p.m. on September 23, 2008 . Note that these are 6 hour before landfall, 7 hours before landfall, 8 hour before landfall, 9 hours before landfall and 10 hours before landfall respectively.

Figure 10, shows the predicted contours of the barometric pressure for 9a.m., September 24, 2008 based on our prediction approach using 6 hour learning, 7 hour learning, 8 hour learning, 9 hour learning, 10 hours learning as well as 11 hour learning.

Figure 6 shows the error of our prediction in Nautical Miles. Each row contains a prediction length in hours from $1,2,\cdots,12$ hours in advance. The columns are the number of learning periods, $L = 1,2,3$.

6 Concluding Remarks

We have extended the Guillas-Lai's Brute-Force Method to study the prediction in the setting where both the explanatory variable and the response variable are real random surfaces. We use it to predict the path of hurricanes. We implemented our extended method to numerically simulate the prediction of Hurricanes Katrina in 2005 and Ike in 2008.

	1 Period	2 Periods	3 Periods
1	29.10	29.10	0.00
2	29.10	29.05	34.52
3	27.00	29.05	27.00
4	58.24	84.45	76.47
5	111.07	60.35	84.45
6	111.07	103.99	84.45
7	101.96	90.43	90.43
8	86.25	80.21	80.21
9	111.07	90.43	101.96
10	111.07	180.65	145.81
11	111.07	145.81	118.71
12	167.10	145.81	118.71

Fig. 6 Errors in Nautical Miles for Hurricane Ike

1. Even though, we only have 80 buoys in the Gulf of Mexico, we are still able to predict the Katrina within 43Nm based on 6 hour lead time and 20 Nm based on 7 hour lead time with 2 learning periods.
2. After Katrina, more buoys were placed in the Gulf of Mexico. Numerical results show that we are able to predict the Ike with more stability.
3. However, the predictions of Ike are not as accurate as the predictions of Katrina. One of reasons is that many of buoys were placed far outside the region of interest for Ike which is close to the coastline near the Texas. Of course, those buoys far outside the Gulf of Mexico will be useful to predict the hurricanes which directly hit the east coast of Florida. Another reason is that the underlying triangulation is much larger than the one for our Katrina prediction study. According to our knowledge of bivariate splines, the approximation will be better when we use a triangulation of smaller size.
4. We should study the prediction of Ike using the triangulation for the study of Katrina to see that the prediction of Ike would be better.
5. Our prediction with 12 hour lead time is not accurate at all. More study is necessary as this will be one of key issues as the 12 hour lead time will give residents enough time to evacuate.
6. Our suggestion to the meteorologists and NOAA is to place more buoys inside the Gulf of Mexico for a better prediction of hurricanes which may hit coastline near the Texas, Louisiana, Alabama, the west coast of Florida. In addition, more buoys are needed to predict more accurately the hurricanes to hit the east coast of Florida.

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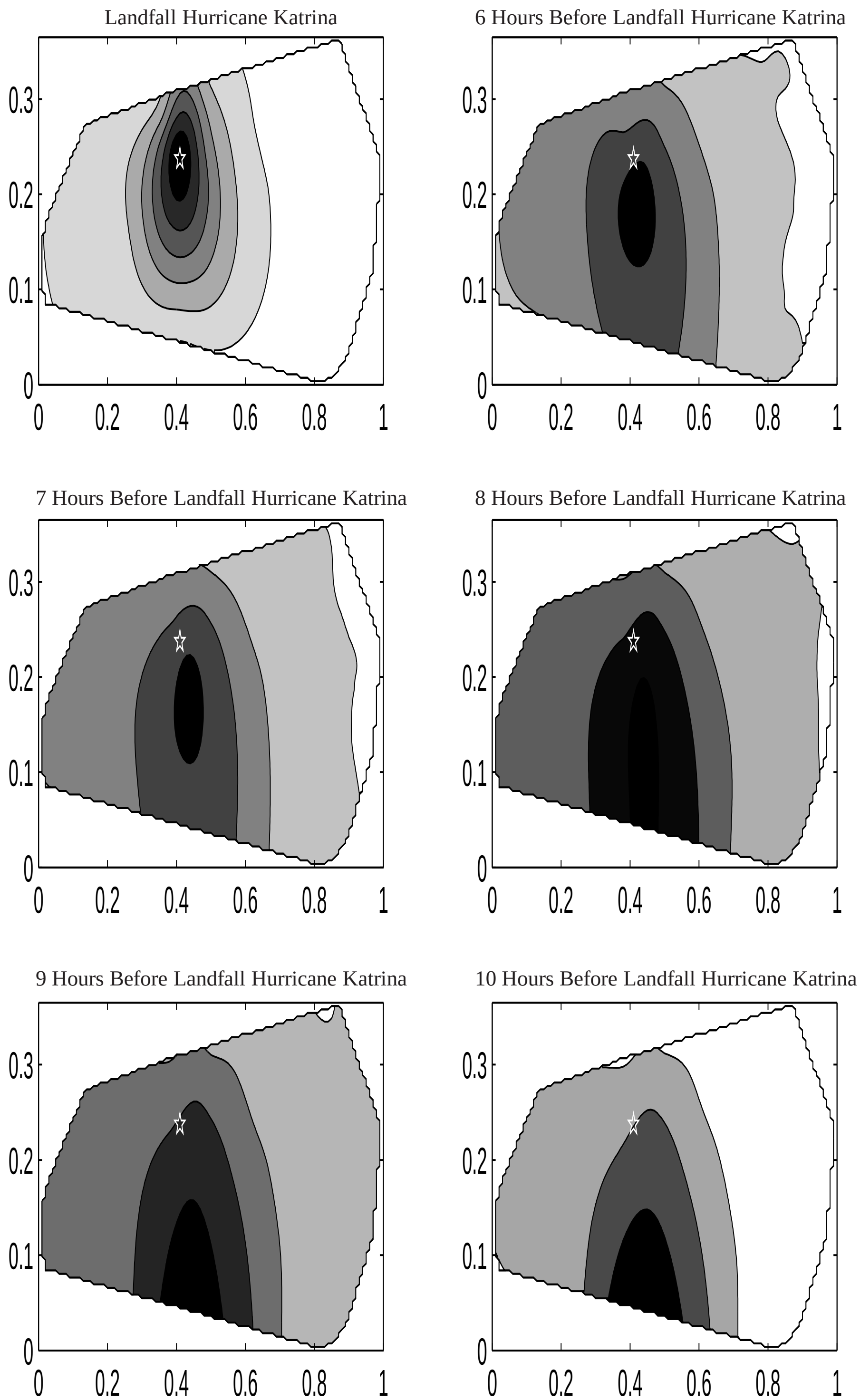

Fig. 7 Exact measurements based on penalized least squares splines

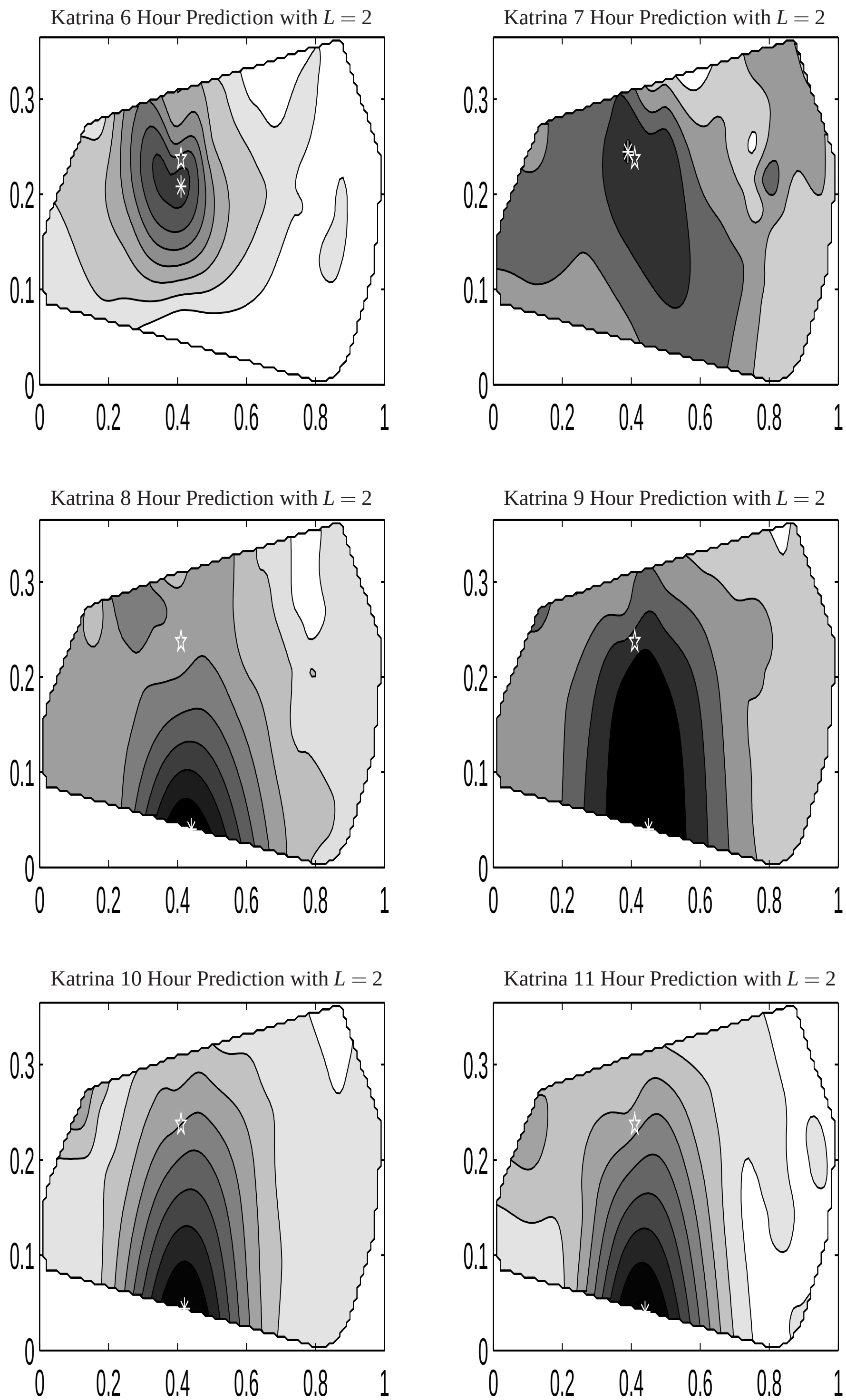

Fig. 8 Predictions for Hurricane Katrina

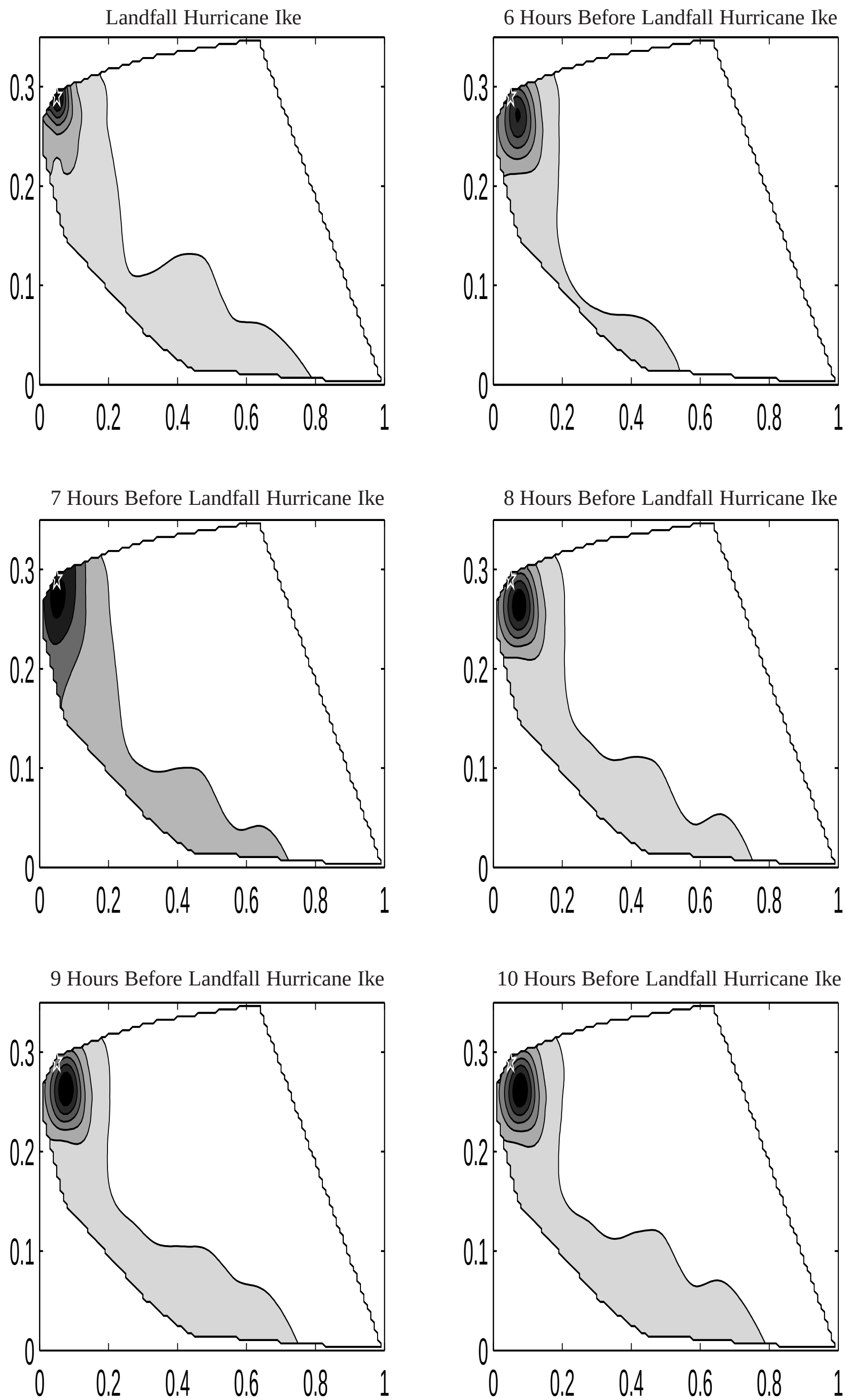

Fig. 9 Exact measurements for Hurricane Ike based on penalized least squares splines

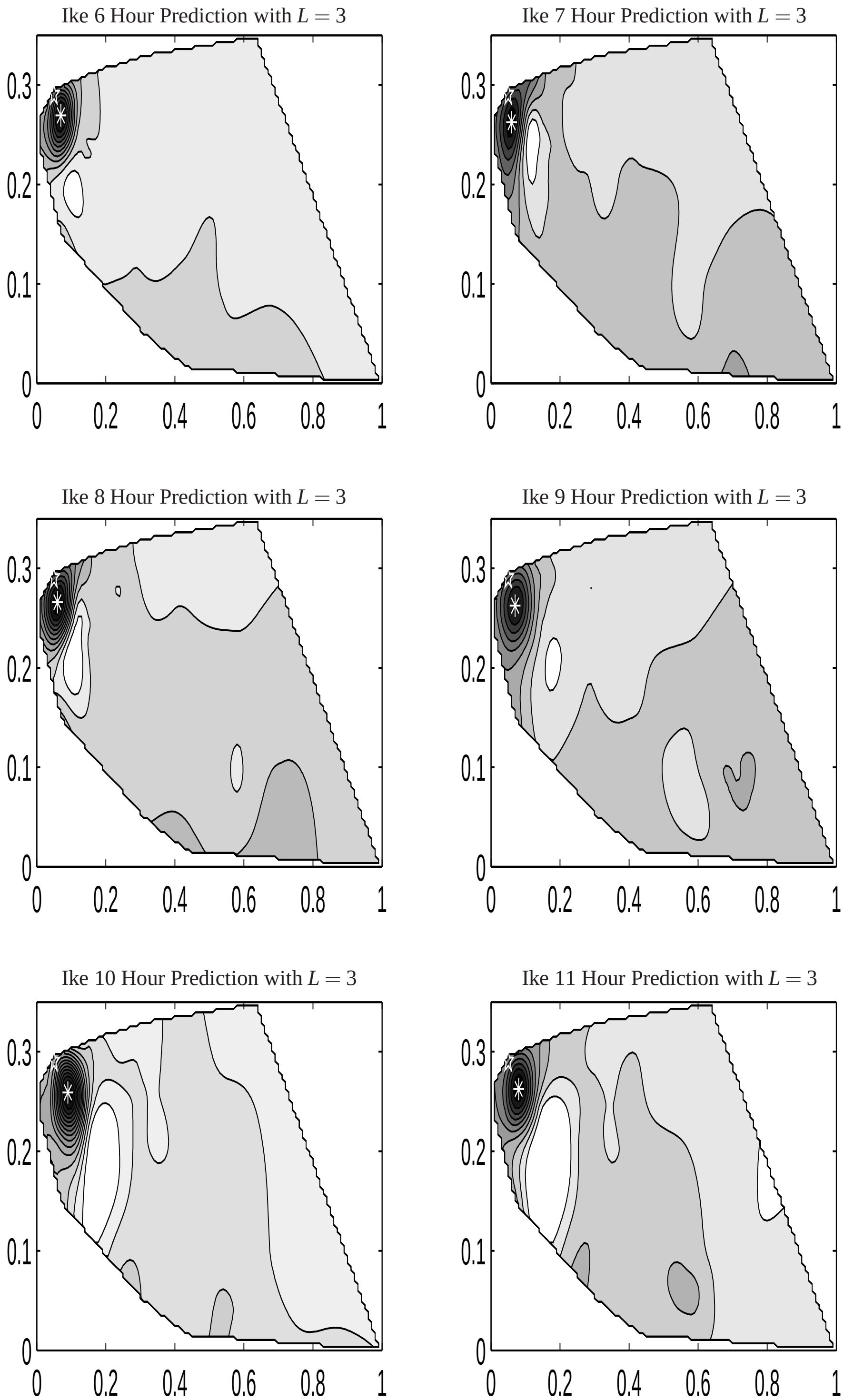

Fig. 10 Predictions for Hurricane Ike