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to evolve in introductory differential equations/dynamical systems courses, this text reflects an emphasis on a thorough understanding of the analytic foundations of the subject. It has enough examples and explanation to satisfy readers for whom a terse “theorem/proof” style is unappealing, while retaining proofs of important theorems.

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Spline Functions on Triangulations. By Ming-Jun Lai and Larry L. Schumaker. Cambridge University Press, Cambridge, UK, 2007. \$148.00. xvi+592 pp., hardcover. ISBN 978-0-521-87592-9.

Splines play a fundamental role in numerical analysis and engineering. Applications include approximation of functions and data, image representation and rendering, computer-aided design and automated manufacturing, as well as the solution of ordinary and partial differential equations. In addition, a number of beautiful theorems in pure mathematics involve splines. Diophantine equations and n -width are perhaps the most prominent examples.

The theory of univariate splines has reached a state of perfection. B-splines serve as an ideal basis with excellent numerical and algorithmic properties. By now, a number of books, including *Spline Functions: Basic Theory* by the second author, published by Wiley in 1981, provide a comprehensive treatment of the theory and its applications.

Toward the end of the last century, multivariate splines were the subject of intensive research. There is, however, no canonical generalization of the univariate theory. The simplest approach using tensor products is often adequate, but lacks the local flexibility familiar from the b-spline calculus. As a consequence, spline functions on triangulations have been widely studied. One motivation is finite element methods, where triangular basis functions have been popular for a long time. Another reason is challenging mathematical problems, even at the very basic level. For example, the dimension of the space of bivariate continuously differentiable piecewise cubic polynomials on general triangulations is still not completely understood.

With more than 1,000 papers on multivariate splines (cf. the online spline bibliography of Carl de Boor and Larry Schumaker at <http://www.math.vanderbilt.edu/~schumake/splinebib.html>), a comprehensive textbook is long overdue. This monograph thus comes just at the right time. It provides researchers, practitioners, and students with a solid foundation for their future work.

The authors begin by briefly reviewing basic properties of bivariate polynomials, including error estimates in the maximum and L_q -norms and the construction of interpolants. Then, the Bernstein–Bézier representation is discussed in detail. Since the pioneering work of Bézier and de Casteljau, the so-called B-form has become an essential tool not only for geometric modeling but also for the analysis of piecewise polynomial spaces. Principal features of the B-form are elegant and efficient algorithms, simple descriptions of smoothness constraints, and shape-preserving properties. As another prerequisite for the analysis of bivariate splines, triangulations are discussed. The authors describe their construction as well as their geometric and combinatorial properties. Special emphasis is given to the Delaunay triangulation with its favorable features.

A major part of the book is devoted to bivariate splines, a topic strongly influenced by the work of the authors and their students. The modern treatment is based on the B-form. A continuous triangular spline

is represented by its Bernstein–Bézier coefficients, the B-net. Splines with higher smoothness can be characterized by minimal determining sets, i.e., subsets of the B-net comprising the free B-coefficients. In the spirit of the familiar b-spline calculus, this approach facilitates the construction of local bases and quasiinterpolants. Moreover, a simple unified treatment of classical C^1 and C^2 macro elements (e.g., the elements introduced by Clough–Tocher, Powell–Sabin, and Wang) can be given. The results have direct applications to data fitting and finite element methods.

The analysis of the bivariate spline space

$$S_d^r(\Delta) = \{p \in C^r : \text{degree } p \leq d \\ \text{on triangles of } \Delta\}$$

for general triangulations Δ is rather delicate, in particular when the smoothness r is high compared to the degree. The authors present the theory in several steps. First, bounds on the dimension in terms of the numbers of vertices and edges are obtained. Exact results are given for $d \geq 3r + 2$ and for type-I and type-II triangulations (partitions of squares, each split by one and two diagonals, respectively). Moreover, the concept of generic dimension is introduced and applied to the intensively studied case of C^1 -cubics. Second, it is shown that the spaces $S_d^r(\Delta)$ have full approximation order for $d \geq 3r + 2$ and suboptimal approximation order for smaller d . Third, the stability of local bases is explored. Again, the restriction on the degree is crucial. Finally, an introduction to box-splines is given with special emphasis on type-I and type-II triangulations. In particular, it is described how to exploit the regular shift invariant structure of box-spline spaces to construct optimal order quasiinterpolants.

The bivariate B-form calculus can also be applied to analyze splines on the sphere. The resulting very elegant theory for piecewise spherical harmonics developed by Alfeld, Neamtu, and Schumaker is described in detail in the book. In particular, algorithms for manipulating the B-form, spherical patches, bases for smooth spherical splines, macro elements, and approximation schemes are discussed. The last topic is considerably more subtle than the planar analogue. To obtain error estimates, a

radial projection mapping onto the tangent planes is utilized, which allows us to apply results for bivariate splines. Spherical splines provide excellent tools for numerical methods in the geosciences, notably for fitting and visualization of spherical data as well as for simulations via partial differential equations.

The last part of the book is devoted to trivariate splines, i.e., piecewise polynomials defined on tetrahedral partitions. As in the bivariate case, the essential prerequisites, trivariate polynomials, their B-form, and triangulations, are discussed. The treatment could be more compact, since the ideas have already been presented in the simpler bivariate setting. The main topics are algorithms for computing with the B-form, differentiation and description of smoothness constraints, elementary approximation schemes and error estimates, general and regular tetrahedral partitions, Euler relations, and Delaunay tetrahedral partitions.

The main features of the theory for bivariate splines carry over to the trivariate setting. In particular, the authors discuss minimal determining sets, dimension, stable local bases, and approximation order. Two key results are the general lower and upper bounds on the dimension and the existence of star-supported bases for $d \geq 8r + 1$. This restriction seems a bit discouraging at first sight. However, smoothness beyond continuity of second derivatives ($r = 2$) is rarely needed in practice. Hence, C^1 and C^2 macro elements, as discussed in the final chapter of the book, provide adequate solutions to most approximation problems. They form a stable basis for the corresponding special trivariate spline spaces and have full approximation order. Four C^1 and five C^2 macro elements are considered in detail, corresponding to different splittings of the macro tetrahedron. A general construction for a C^r macro element is also given.

Summarizing, the book by Ming-Jun Lai and Larry Schumaker gives a comprehensive treatment of bivariate, spherical, and trivariate splines on triangulations. It covers all essential theoretical and algorithmic aspects and provides a solid foundation for future work. Researchers interested in multivariate splines will consider it an excellent

reference text, and applied mathematicians as well as engineers will find tools for applications in data fitting, geometric modeling, visualization, and finite element analysis. With a proper selection of the material, the book could also serve as a text for graduate students. The authors take great care to make the material also easily accessible to the nonexpert. For the reviewer, the monograph has been a pleasure to read.

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A Primer for Mathematics Competitions.

By A. Zawaira and G. Hitchcock. Oxford University Press, Oxford, 2008. \$45.00. xviii+344 pp., softcover. ISBN 978-0199-5398-88.¹

Basic Introduction. As its title suggests, this is a book to help prepare students for mathematical competitions. It does this successfully by providing coverage of the usual topics to be found in most school mathematical competitions up to the International Mathematical Olympiad, as well as providing practice in solving related problems. This preparation, though, does rely on the reader doing some hard work. It is not a book to just be read. The full value of the volume can only be gained by trying to solve the problems that are provided.

This is a book both for bright students to work through by themselves and for teachers to enable them to understand what is required in competitions so that they can help their students prepare for such events. It is also a ready store of results and methods that can be dipped into when needed.

Format. The book starts with a preface in which the authors answer such questions as, what is a Mathematics Olympiad; why do young people enter Mathematics

Olympiads; and what benefits does training for a Mathematical Olympiad bring.

Then follow nine chapters that the authors call toolchests. The first eight of these cover the basic topics that are to be found in most mathematics competitions for school students: geometry; inequalities and induction; Diophantine equations; number theory; trigonometry; sequences and series; the binomial theorem; and counting techniques. The final toolchest is a collection of miscellaneous problems and their solutions. These problems do not always rely on aspects of the ideas introduced in earlier chapters for their solution.

Roughly speaking, the first eight toolchests begin with a statement of what the reader should be able to do after working through that chapter. That is followed by “Appetizer Problems” and various sections that take up a particular new idea or result, and these are exemplified by worked examples. The final two sections always contain problems whose solutions require material from the chapter and the solutions of those problems.

Why I Like It. I think that the first reason that this book appeals to me is that it is written in an interesting way. The authors go out of their way to engage the reader. They do this in a number of ways that include, in no particular order, providing history and background to the material they are presenting, whetting the reader’s appetite for a chapter by providing “Appetizers,” emphasizing the need for proof in both competitions and mathematics itself, and providing a significant number of cartoons throughout (though note my caveat on cartoons later).

Pieces of mathematical history are peppered throughout the text, usually at the start of new topics. So we hear about the early mathematics of the Greeks and Babylonians, a development of number from real to irrational to complex, as well as results on geometry that are as recent as 2004.

The “Appetizers” seem to me to be a good idea for challenging the reader and providing encouragement to master the techniques of a chapter or section.

The eight topics that are taught in this book are the ones that seem to be in every

¹When reading this review it should be noted that its author has just published *A First Step to Mathematical Olympiad Problems*, World Scientific, Singapore, 2009. His book is based on the material described in the reviewed book on page 337f. The address quoted in the reviewed book may no longer be current. The two books cover the same material and address the same audience.