Syllabus for MATH8550

Spring, 2010, Dr. Ming-Jun Lai

Overview

Optimization is a very useful tool for applied mathematics. It has found applications in many areas of mathematics and applied sciences including Economics and Operational Research. I plan to motivate our study by first examining five areas of applications which use optimization approaches: scattered data interpolation/fitting, statistical applications, numerical solution of partial differential equations, data compression, image processing such as de-noising and de-blurring. Then I will explain classic theory for optimization by starting with convex sets, convex functions, convex functionals, partial differentiation, Gateaux differentiation, Frechet derivatives, sub-differentiation and then presenting an analysis for unconstrained minimization, e.g. the best approximation and constrained minimization, e.g., Lagrange multiplier method and Kuhn-Tucker conditions, linear and convex programmings. In addition, I will present a duality approach by converting the minimization problem into a maximization problem. Finally, I will discuss several classic numerical methods for optimization, e.g., Newton method, Steepest Descent method, and Conjugated Gradient Method. These form the first half of the semester. I plan to spend the second half of the semester discussing the modern theory of optimization with emphasis on L_1 approximation and minimization. Several brand new approaches will be presented including projected gradient methods, Bragmen iterative algorithms, the basis pursuit method, Orthogonal Greedy algorithms, the ℓ_q minimization and etc..

If you want to see me for questions, please make an appointment right after our class or email me. There will be a final project or final examination for this class dependent on your preference. Many homework problems will be assigned during the class.

Tentative Schedule

1/8 Introduction

1/11 Optimization for Scattered Data Fitting

1/13 Optimization for Data Fitting(II)

1/15 Optimization for Statistical Applications

1/18 No class

1/20 Optimization for Numerical Solution of PDE

1/22 Optimization for Numerical Solution of PDE(II)

1/25 Optimization for Numerical Solution of PDE(III)

1/27 Optimization for Image Denoising

1/27 Optimization for Image Deblurring

2/1 Optimization for Compressed Sensing

2/3 Optimization for Compressed Sensing(II)

2/5 Classic Theory of Convex Analysis(I)

2/8 Classic Theory of Convex Analysis (II)

2/10 Classic Theory of Optimization (I)

2/12 Classic Theory of Optimization (II)

2/15 Best Approximation in Hilbert Spaces

2/17 Best Approximation in L^1 norm

2/19 Best Approximation in Convex Functionals

2/22 Duality Approach

2/24 Conjugate Functionals

2/26 Newton's Method

3/1 Steepest Descent Method

3/3 Conjugate Gradient Method

3/5 Uzawa Iterative Method

3/8–3/12 Spring Break

3/15 ℓ^1 minimization

3/17 The Simplex Method

3/19 The Interior Point Method

3/22 Restricted Isometry Property

3/24 Orthogonal Greedy Algorithm

3/26 **Final Project Assignment**

3/29 Bregman Iterative Method(I)

3/31 Bregman Iterative Method(II)

4/2 Linearizations

4/5 Projected Gradient Method

4/7 Projected Gradient Method(II)

4/9 Generalization of PGM

4/12 A Minimal Surface Area Approach

4/14 Convergence of Iterative Method

4/16 The ℓ^q minimization

4/19 MMV and computational algorithm

4/21 Unconstrained ℓ^q minimization

4/23 Parseval Expansion

4/26 Parseval Expansion(II)

4/28 Parseval Expansion(III)

4/29 **Review For Final Examination**