Date: 17 September 2009
Section: 4.1: **Inverse Functions**

Two functions $f(x)$ and $g(x)$ are said to be inverses if their mutual compositions equal to x. That is,

$$f(g(x)) = g(f(x)) = x.$$

If this holds, $f(x)$ and $g(x)$ are said to be **inverses** of each other. I will explain in class what this entails graphically.

A function $f(x)$ is said to be **invertible** or **one to one** if it has an inverse function. Graphically, if a function $f(x)$ is invertible, then it passes the **horizontal line test.**

Properties of Inverse Functions

If $f(x)$ and $g(x)$ are inverse functions, then we have the following properties:

1. The domain of $f(x)$ is the range of $g(x)$.
2. The range of $f(x)$ is the domain of $g(x)$
3. The graphs of $f(x)$ and $g(x)$ are mirror images of each other over the line $y = x$.

Moreover, if $f(x)$ is invertible, then its inverse is usually denoted as $f^{-1}(x)$. Note that this is **not** the same thing as the reciprocal!

Computing f^{-1} Algebraically.
Here's an example of computing for inverses: Let $f(x) = 3x - 1$. Do the following:

1. Using your graphing calculator, determine if $f(x)$ passes the horizontal line test.
2. Now let's look for $f^{-1}(x)$. Since $f(x)$ and $f^{-1}(x)$ are inverses, the domain of $f(x)$ is the range of $f^{-1}(x)$. This means in our current equation, we can switch $f(x)$ with x and switch x with $f^{-1}(x)$. This gives us that:

   $$f(x) = 3x - 1 \qquad \longrightarrow \qquad x = 3f^{-1}(x) - 1$$

   We then solve for $f^{-1}(x)$.
3. Use algebra to come up with

   $$f^{-1}(x) = \frac{x+1}{3}$$

HARDER EXAMPLE: Given that $f(x)$ is defined below, compute $f^{-1}(x)$:

$$f(x) = \frac{3x-4}{2x+1}$$

$$\begin{aligned}
f(x) &= \frac{3x-4}{2x+1} && \text{Given problem.} \\
x &= \frac{3f^{-1}(x)-4}{2f^{-1}(x)+1} && \text{Do your switch.} \\
x(2f^{-1}(x)+1) &= 3f^{-1}(x)-4 && \text{Cross multiply.} \\
2xf^{-1}(x)+x &= 3f^{-1}(x)-4 && \text{Distribute.} \\
2xf^{-1}(x)-3f^{-1}(x) &= -4-x && \text{Gather your } f^{-1}(x) \text{ terms to one side.} \\
2x-3\left(f^{-1}(x)\right) &= -4-x && \text{Factor out your } f^{-1}(x). \\
f^{-1}(x) &= \frac{-4-x}{2x-3} && \text{Isolate } f^{-1} \text{ by dividing.}
\end{aligned}$$