

PRACTICE PROBLEMS FOR 2200 MIDTERM EXAM 1

PETE L. CLARK

- 1) Give the definition of the derivative of a function f at the point x .
- 2) Give an equation for the tangent line to $y = f(x)$ at the point $x = a$.
- 3) Give a definition (in terms of limits) of continuity of f at the point x .
- 4) Give a definition of “removable discontinuity”.
- 5) State the Intermediate Value Theorem.
- 6) For each of the following, mark as true or false, and give brief explanations.
 - a) A polynomial function $P(x)$ is continuous at every point at which it is defined.
 - b) A rational function $\frac{P(x)}{Q(x)}$ is continuous at every point at which it is defined.
 - c) For every rational function $\frac{P(x)}{Q(x)}$, $\lim_{x \rightarrow \infty} \frac{P(x)}{Q(x)}$ exists.
 - d) For every rational function $\frac{P(x)}{Q(x)}$ for which the numerator and the denominator are polynomials of the same degree, $\lim_{x \rightarrow \infty} \frac{P(x)}{Q(x)}$ exists and is nonzero.
 - e) If rational function $\frac{P(x)}{Q(x)}$ has a vertical asymptote at $x = c$, then $Q(c) = 0$.
 - f) If $Q(c) = 0$, then the rational function $\frac{P(x)}{Q(x)}$ has a vertical asymptote at $x = c$.
 - g) It is possible for a function to be differentiable at x but not be continuous at x .
 - h) It is possible for a function to be continuous at x but not differentiable at x .
- 7) Let $y = f(x) = 7x^2 + 5x - 11$.
 - a) Compute the derivative y' and the second derivative y'' .
 - b) Compute the equation of the tangent line to $y = f(x)$ at a general point $x = a$.
 - c) Find all points a such that the tangent line to $y = f(x)$ is horizontal.
- 8) Evaluate the following limits (or show that they do not exist):
 - a) $\lim_{x \rightarrow 2} \frac{x^2 4x - 12}{x - 2}$.
 - b) $\lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$.
 - c) $\lim_{x \rightarrow 0} \frac{\sin 3x(2 - 2 \cos x)}{\sin 11x}$.
- 9) For which values of x are the following functions continuous?
 - a) $f(x) = \frac{4x}{\sqrt{49 - x^2}}$.
 - b) $f(x) = (x - 8)^{1/3}$.
 - c) $f(x) = |\sin x|$.

10) Find an integer n such that the polynomial $f(x) = x^4 - 4x^3 + 2x^2 + 2$ has at least one root c with $n \leq c \leq n + 1$.

11) Show that there exists a real number x such that $2^x = \cos x$.

12) Sketch a graph of each of the following functions.

a) $f(x) = \sin x$.

b) $f(x) = |\sin x|$.

c) $f(x) = x \sin x$.

d) $f(x) = |x \sin x|$.