

PROF. CLARK'S MATH 3200 FALL 2009 MIDTERM 1

Part I: Do all of the following problems.

I.1) a) Show that the following are logically equivalent:

(i)  $A \implies (B \vee C)$ ;

(ii)  $(A \wedge \sim B) \implies C$ .

b) Show that for all  $x \in \mathbb{Z}$ ,  $x(x^2 + 1)$  is even.

I.2) Negate the following statements:

a)  $x$  is odd and if  $y$  is even, then  $z$  is odd.

b) For any line  $\ell$  in the plane and any point  $P$  not lying on  $\ell$ , there exists exactly one line  $\ell'$  passing through  $P$  and parallel to  $\ell$ .

c) I'm not going to lower my voice, and I'm staying right where I am.

d) All's well that ends well.

I.3) Consider an implication of the form " $\forall x \in S, P(x) \implies Q(x)$ ."

a) What does it mean for the implication to hold trivially?

b) What does it mean for the implication to hold vacuously?

c) Let  $S = \mathbb{Z}$ . Let  $P(x)$  be " $1298548x + 1509850980$  is odd", and let  $Q(x)$  be " $191 \mid x^4 + 53x^3 + 17x^2 - 14$ ". Show that for all  $x \in S$ ,  $P(x) \implies Q(x)$ .

d) Let  $S$  be the set of rational numbers. Let  $P(x)$  be " $e^x$  is an irrational number." Let  $Q(x)$  be " $x^4 + 1 \geq 2x^2$ ". Show that for all  $x \in S$ ,  $P(x) \implies Q(x)$ .

I.4) Consider an implication of the form " $\forall x \in S, P(x) \implies Q(x)$ ."

a) What does it mean for the implication to hold trivially?

b) What does it mean for the implication to hold vacuously?

c) Let  $S = \mathbb{Z}$ . Let  $P(x)$  be " $1298548x + 1509850980$  is odd", and let  $Q(x)$  be " $191 \mid x^4 + 53x^3 + 17x^2 - 14$ ". Show that for all  $x \in S$ ,  $P(x) \implies Q(x)$ .

d) Let  $S$  be the set of rational numbers. Let  $P(x)$  be " $e^x$  is an irrational number". Let  $Q(x)$  be " $x^4 + 1 \geq 2x^2$ ". Show that for all  $x \in S$ ,  $P(x) \implies Q(x)$ .

Part II: Do **any two of the following three** problems.<sup>1</sup>

II.1) Prove or disprove:

a) If  $x, y, z$  are objects such that  $x \in y$  and  $y \in z$ , then  $x \in z$ .

b) If  $X, Y, Z$  are sets such that  $X \subseteq Y$  and  $Y \subseteq Z$ , then  $X \subseteq Z$ .

c) If  $X, Y, Z$  are sets such that  $X \subsetneq Y$  and  $Y \subsetneq Z$ , then  $X \subsetneq Z$ .

II.2) Let  $A$  and  $B$  be sets. Show that  $A \cap B = A \cup B$  if and only if  $A = B$ .

II.3) Show that for all  $x \in \mathbb{Z}$ ,  $8 \mid (x - 1)(x - 2)(x - 3)(x - 4)$ .

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<sup>1</sup>If it is not clearly indicated which two I am meant to grade, I will simply grade the first two problems that have nonempty answers.