AMERICAN MATHEMATICAL SOCIETY
MathSciNet Mathematical Reviews on the Web

Previous | Up | Next Article

Citations
From References: 2
From Reviews: 3

MR993219 (90e:11085) 11G05 (11D09 14K07 14K15)

Müller, Hans H. [Müller, Hans Helmut] (D-SAAR); **Ströher, Harald** (D-SAAR);
Zimmer, Horst G. (D-SAAR)

Torsion groups of elliptic curves with integral j-invariant over quadratic fields.

J. Reine Angew. Math. **397** (1989), 100–161.

Let K be a quadratic number field $\mathbf{Q}(\sqrt{D})$. The authors show: If an elliptic curve E defined over K has integral j-invariant, then the torsion group $E(K)_{\rm tor}$ of E is isomorphic to one of the following groups: $\mathbf{Z}/N\mathbf{Z}$ $(1 \le N \le 8,\ N = 10)$, $\mathbf{Z}/2\mathbf{Z} \oplus \mathbf{Z}/2n\mathbf{Z}$ $(n = 1, 2, 3)$, $(\mathbf{Z}/3\mathbf{Z})^2$. Further, they show that these groups do occur and except for the cases $E(K)_{\rm tor} \cong \mathbf{Z}/N\mathbf{Z}$ $(N = 2, 3)$, $(\mathbf{Z}/2\mathbf{Z})^2$, the corresponding E and K are finite in number. (These corresponding E and K are completely determined.) The essential part of the proof is to give parametrizations and characterizations of the j-invariants of the elliptic curves with torsion group of a given type. This is done by an elementary but tedious case-by-case discussion. From this, the problem is reduced to solving the Diophantine equations of the type: $X^2 - DY^2 = a$ $(a \in \mathbf{Z},\ X, Y \in \frac{1}{2}\mathbf{Z})$. See also the conjecture of M. A. Kenku and F. Momose [Nagoya Math. J. **109** (1988), 125–149; MR0931956 (89c:11091)] for the possible types of $E(K)_{\rm tor}$.

Reviewed by *Noburo Ishii*

© *Copyright American Mathematical Society 1990, 2007*