



Supplement to Numerical Computations Concerning the ERH

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Supplement to
NUMERICAL COMPUTATIONS CONCERNING THE ERH
ROBERT RUMELY

Section 5. Tables and Graphs.

Table 5.1: STATISTICS ABOUT L-SERIES, BY MODULUS

#	ERH	SUM	LEAST	LEAST	GREATEST	GREATEST	LEAST	GREATEST	
Q	L-SER	HEIGHT	1/ R	ROOT	MAX	MAX	S(T)	GAP	GAP
			BOUND						
1	1	10029.021	4.322	14.13473	0.003967	16.90615	1.44847	0.04210	2.48351
3	1	10133.702	5.672	8.03974	0.004529	15.04223	1.34760	0.05368	2.59871
4	1	10248.864	6.092	6.02095	0.001965	11.86942	1.37010	0.03333	2.52035
5	3	10072.973	19.100	4.13290	0.001475	19.62612	1.48385	0.03512	2.62789
7	5	10017.500	34.278	2.50937	0.000316	23.50929	1.52513	0.01357	2.75128
8	2	10021.581	14.056	3.57615	0.002663	13.59221	1.37205	0.04961	2.48191
9	4	10157.746	28.916	2.90199	0.000869	18.22939	1.47636	0.02545	2.70072
11	9	10019.363	68.002	1.23119	0.000502	27.66859	1.61542	0.02522	2.81486
12	1	10026.176	7.573	3.80463	0.001977	9.30459	1.28586	0.03403	2.41170
13	11	10019.119	86.079	0.88396	0.000096	28.54738	1.65258	0.00899	2.83365
15	3	2524.086	17.654	2.73460	0.000932	13.13095	1.29729	0.02518	2.22265
16	4	2520.832	24.138	1.58558	0.002081	11.48749	1.33572	0.03984	2.42998
17	15	2748.166	94.573	0.39131	0.000748	23.89895	1.61490	0.02081	2.69313
19	17	2512.243	107.915	0.01896	0.002810	23.24676	1.57042	0.04142	2.82688
20	3	2604.865	18.965	2.35893	0.006685	9.64142	1.28374	0.05928	2.22985
21	5	2544.293	31.877	1.61202	0.001400	14.41247	1.37781	0.02660	2.47479
23	21	2542.055	138.599	0.59543	0.001209	24.61406	1.57408	0.03103	2.79958
24	2	2612.653	13.220	1.97719	0.001093	7.96155	1.26778	0.02606	2.15096
25	16	2506.712	106.934	0.39645	0.000123	21.44001	1.47188	0.01258	2.60182
27	12	2541.274	81.797	0.40516	0.000391	17.43803	1.47291	0.02043	2.65296
28	5	2503.605	33.848	1.20418	0.001316	11.61433	1.32874	0.02480	2.40888
29	27	2512.351	186.749	0.28629	0.001480	25.70754	1.60159	0.02479	2.84417
31	29	2515.510	203.685	0.00694	0.000809	25.58984	1.63640	0.02346	2.80325
32	8	2646.407	57.191	0.33600	0.002206	13.37724	1.46624	0.04030	2.48466
33	9	2530.468	63.576	0.50072	0.001538	16.73714	1.40489	0.02966	2.46203
35	15	2510.826	106.872	0.58660	0.000182	20.25735	1.43655	0.00878	2.53116
36	4	2585.494	28.858	0.93469	0.002588	9.34404	1.25067	0.03808	2.34660
37	35	2520.072	254.727	0.12353	0.001259	28.98442	1.61969	0.03535	2.93011
39	11	2524.507	80.053	0.65151	0.001633	18.52665	1.41161	0.02675	2.58028
40	6	2586.457	44.257	0.78176	0.006153	11.37038	1.32871	0.05905	2.44113
41	39	2505.993	289.095	0.08543	0.000290	27.46621	1.61801	0.01642	2.84475
43	41	2505.223	306.837	0.13837	0.000026	27.88252	1.59767	0.00472	2.83346
44	9	2535.143	67.753	0.12372	0.000830	12.58658	1.36247	0.01587	2.49687
45	12	2517.289	90.410	0.14131	0.000297	15.32316	1.43865	0.01316	2.49948
47	45	2506.052	342.626	0.09038	0.000517	29.20429	1.64085	0.01762	2.88328
48	4	2521.707	30.553	0.60932	0.006073	9.27006	1.29900	0.04740	2.36446
49	36	2508.798	275.753	0.09468	0.000230	26.84519	1.62201	0.01366	2.74264
51	15	2526.701	115.205	0.56066	0.001197	18.69700	1.53718	0.02985	2.56331
52	11	2553.836	84.952	0.48615	0.000310	13.38493	1.41161	0.01423	2.46340
53	51	2548.480	398.252	0.00643	0.000399	29.53259	1.67729	0.02156	2.96062
55	27	2509.998	210.645	0.13205	0.002016	22.27891	1.56915	0.03851	2.63395
56	10	2516.789	78.323	0.52630	0.000730	12.70161	1.36402	0.02221	2.44271
57	17	2509.169	133.213	0.31033	0.000356	19.79805	1.55137	0.01533	2.75323
59	57	2505.503	452.691	0.05165	0.000126	29.42993	1.74724	0.00986	2.94160
60	3	2513.720	23.761	0.93760	0.007494	7.45733	1.27038	0.05993	2.24913
61	59	2512.627	471.598	0.02825	0.000109	29.98239	1.68757	0.01400	3.03949
63	20	2503.432	160.015	0.21101	0.000553	17.34584	1.43459	0.01819	2.51445
64	16	2508.451	128.467	0.37696	0.000095	15.06637	1.51673	0.00851	2.74952
65	33	2516.155	265.591	0.18416	0.000867	23.68456	1.59941	0.02218	2.72152
67	65	2506.077	328.090	0.11303	0.000263	30.46775	1.75660	0.01495	2.96521
68	15	2506.422	121.598	0.05346	0.001380	14.93672	1.47440	0.03542	2.55268
69	21	2512.006	170.883	0.22273	0.003124	19.76238	1.56791	0.04035	2.65577
71	69	2515.050	566.761	0.09859	0.000353	31.24616	1.68277	0.01463	2.97862
72	8	2614.933	66.205	0.27916	0.002330	9.81096	1.39838	0.03341	2.44542

Table 5.1 (Cont.): STATISTICS ABOUT L-SERIES, BY MODULUS

Q	L-SERIES	ERR HEIGHT	SUM 1/R	LEAST ROOT	LEAST MAX	GREATEST	GREATEST GAP	LEAST GAP	GREATEST GAP
75	16	2529.460	132.413	0.000000	16.78104	1.42792	0.02480	2.57318	
76	17	2505.068	140.602	0.000016	15.20326	1.71011	0.00152	2.62377	
77	45	2505.664	373.562	0.000014	25.86384	1.00131	0.00131	2.76008	
80	12	2511.333	101.597	0.000000	12.66968	1.00196	0.04196	2.42879	
81	36	2511.668	301.428	0.000017	21.89038	1.00112	0.00112	2.28930	
84	5	2512.153	62.005	0.000000	6.64278	1.00000	0.00000	2.28939	
85	45	2512.244	259.495	0.000022	26.55321	1.00000	0.00000	2.48439	
87	27	2512.539	152.635	0.000018	12.19910	1.00000	0.00000	2.65824	
88	18	2512.539	152.635	0.000018	12.19910	1.00000	0.00000	2.65824	
91	55	2512.539	152.635	0.000018	12.19910	1.00000	0.00000	2.65824	
92	21	2512.429	149.885	0.000000	15.91243	1.00000	0.00000	2.78921	
93	29	2512.203	248.072	0.000025	21.55503	1.00000	0.00000	2.56604	
95	51	2512.712	438.298	0.000034	26.90276	1.00000	0.00000	2.74894	
96	8	2513.747	68.584	0.000013	10.68359	1.00000	0.00000	2.49791	
99	36	2513.606	319.027	0.000049	21.39498	1.54025	0.02345	2.70441	
100	16	2513.335	132.050	0.000014	13.25361	1.48805	0.00000	2.60172	
104	22	2513.853	192.814	0.000219	15.30344	1.47779	0.00000	2.74152	
105	15	2513.713	131.942	0.000161	15.71932	1.42012	0.00000	2.48152	
108	12	2513.200	105.636	0.000132	10.21365	1.51110	0.00000	2.45700	
111	35	2513.740	177.399	0.000553	14.07386	1.45388	0.00000	2.37117	
112	20	2513.642	240.304	0.000600	16.60488	1.54730	0.00000	2.72823	
119	46	2513.900	392.705	0.000313	22.02380	1.51906	0.00000	2.45700	
120	100	2513.000	897.144	0.000172	8.31769	1.27648	0.00000	2.46601	
121	100	2513.000	897.144	0.000172	8.31769	1.27648	0.00000	2.46601	
124	29	2513.360	260.710	0.000000	31.26244	1.72166	0.00000	2.73119	
125	80	2510.799	289.578	0.000000	16.78331	1.44015	0.00000	2.69185	
128	32	2505.179	289.578	0.000000	16.78331	1.52485	0.00000	2.72001	
132	9	2513.920	81.745	0.000000	8.31769	1.34672	0.00000	2.46601	
140	15	2513.469	138.254	0.000000	11.33315	1.67142	0.00000	2.46601	
143	99	2513.355	911.957	0.000000	11.33315	1.51110	0.00000	2.46601	
144	16	2513.320	147.599	0.000000	11.33315	1.51110	0.00000	2.46601	
156	11	2513.002	102.925	0.000000	10.21365	1.40271	0.00000	2.46601	
163	161	2513.024	1515.771	0.000000	10.21365	1.40271	0.00000	2.46601	
168	10	2513.277	94.186	0.000000	10.21365	1.40271	0.00000	2.46601	
189	144	2513.534	1362.403	0.000000	10.21365	1.40271	0.00000	2.46601	
180	12	2513.959	114.467	0.000000	10.21365	1.40271	0.00000	2.46601	
216	24	2513.764	235.702	0.000000	10.21365	1.40271	0.00000	2.46601	
246	186	2513.784	1078.863	0.000000	10.21365	1.40271	0.00000	2.46601	
256	168	2513.776	943.989	0.000000	10.21365	1.40271	0.00000	2.46601	
360	24	2513.628	255.117	0.000000	10.21365	1.40271	0.00000	2.46601	
420	15	2504.628	631.117	0.000000	10.21365	1.40271	0.00000	2.46601	
432	48	2509.744	519.392	0.02054	12.89282	1.46812	0.00000	2.32022	

Notes
 (1) The values given are extrema over all L-series with conductor Q. ERR HEIGHT and SUM 1/R: BOUND were computed using interval arithmetic, and are mathematically rigorous, though not sharp. Other entries in the table are believed accurate to the last digit but are not rigorous.
 (2) ERR HEIGHT is a lower bound for the height to which the ERH has been proved uniformly for all L-series with conductor Q.
 (3) SUM 1/R: BOUND is an upper bound for the sum over all L-series with conductor Q of the sums of the reciprocal moduli of the roots $0.5+it$ with $0 < t \leq$ ERR HEIGHT.
 (4) LEAST ROOT is the least T_0 for which $0.5+it$ is a root.
 (5) GREATEST MAX and GREATEST MIN are the smallest and largest maxima between roots.
 (6) GREATEST S(T) is the largest value of $S(T)$ encountered.
 (7) LEAST GAP and GREATEST GAP are gaps between consecutive roots, on a Gram point scale

Table 5.2: CHAMPIONS

SMALL GAPS:										LARGE GAPS:									
Q	K	(GRAM) GAP LENGTH	LOW	HIGH	T	Q	K	(GRAM) GAP LENGTH	LOW	HIGH	T	Q	K	(GRAM) GAP LENGTH	LOW	HIGH	T		
95	8	0.001831	357.3101	1.754738	163	71	3	1.647799	2376.7616	4.771501	50142								
77	4	0.004341	815.1071	6.41037	163	56	3	3.910178	1525.5214	1.9713672	36782								
169	104	0.004689	810.1104	6.41037	163	56	3	3.910178	1525.5214	1.9713672	36782								
43	19	0.004719	1843.8981	13.118218	61	13	3.167752	1148.3243	1.7503655	67487									
111	25	0.005684	1429.2120	11.23673	71	68	2.975627	1218.1938	2.6013930	260330									
163	118	0.007289	272.1143	2.46947	163	46	2.766116	2203.3813	2.6013930	260330									
64	11	0.008754	2348.3232	17.22572	121	16	2.766116	1307.1111	1.9147001	47001									
13	10	0.008990	1516.1279	11.43186	67	16	2.766116	832.4012	1.9147001	47001									
35	0	0.008990	7958.4949	63.68512	169	9	2.766116	1856.2718	1.9147001	47001									
55	0	0.008955	467.5724	6.97972	163	116	2.766116	684.0000	1.9147001	47001									
88	8	0.010521	967.5724	6.97972	163	36	2.766116	1490.3717	1.9147001	47001									
127	46	0.012283	1069.1796	17.861452	169	120	2.766116	1873.1542	1.9147001	47001									
243	63	0.012283	1069.1796	17.861452	169	57	2.766116	2109.1462	2.1112308	35150									
169	12	0.014389	322.0671	9.253666	71	57	2.766116	2495.7616	2.1112308	35150									
169	12	0.014389	322.0671	9.253666	71	57	2.766116	2495.7616	2.1112308	35150									

SMALL CONSECUTIVE PAIRS OF GAPS:										LARGE S(T) (given with sign):									
Q	K	(GRAM) GAP LENGTH	LOW	MIDDLE	HIGH	T	Q	K	S(T)	Q	K	S(T)	Q	K	S(T)	T			
121	99	0.773990	1766.6081	1766.6081	1766.6081	1766.6081	163	105	-1.79084	1513.0310	163	105	-1.79084	1513.0310	163	105	-1.79084		
93	4	0.812268	2414.3701	2444.3701	2444.3701	2444.3701	163	9	1.76726	2120.05764	163	9	1.76726	2120.05764	163	9	1.76726		
111	35	0.812268	2414.3701	2444.3701	2444.3701	2444.3701	163	42	-1.75660	2016.05764	163	101	-1.74983	2467.05116	163	101	-1.74983		
163	105	0.86321	1513.09463	1513.73471	1513.73471	1513.73471	163	59	32	1.74724	1723.05116	163	59	32	1.74724	1723.05116			
163	94	0.86321	1513.09463	1513.73471	1513.73471	1513.73471	163	94	0.86321	1513.09463	1513.73471	1513.73471	163	94	0.86321	1513.09463			
123	94	0.86321	1513.09463	1513.73471	1513.73471	1513.73471	163	126	0.812268	1994.18577	994.83351	163	126	0.812268	1994.18577	994.83351			
100	13	0.812268	1994.18577	1994.18577	1994.18577	1994.18577	163	126	0.812268	1994.18577	1994.18577	163	126	0.812268	1994.18577	1994.18577			
163	126	0.812268	1994.18577	1994.18577	1994.18577	1994.18577	163	126	0.812268	1994.18577	1994.18577	163	126	0.812268	1994.18577	1994.18577			
29	17	0.248150	1029.8873	1029.8873	1029.8873	1029.8873	243	92	0.405150	2379.45115	163	44	1.76726	2016.05764	163	44	1.76726		
243	92	0.405150	2379.45115	2379.45115	2379.45115	2379.45115	256	63	0.11453	2335.12715	163	36	1.1418	1800.36771	163	36	1.1418		
256	63	0.11453	2335.12715	2335.12715	2335.12715	2335.12715	169	117	0.11453	2335.12715	2335.12715	169	117	0.11453	2335.12715	2335.12715			
169	117	0.11453	2335.12715	2335.12715	2335.12715	2335.12715	169	117	0.11453	2335.12715	2335.12715	169	117	0.11453	2335.12715	2335.12715			

SMALL MAXIMA: T: [Z(T,X)]
 LARGE MAXIMA: T: [Z(T,X)]

Table 5.4: STATISTICS ABOUT INDIVIDUAL L-SERIES, THROUGH MODELS 13

CHAR ORDER	ERR HEIGHT	# ROOTS	SUM 1 R ROUND	LEAST ROOT	GREATEST MAX	GREATEST S(T)	LEAST GAP
1	10029.021	10177	4.322	14.13473	16.90615	1.34847	0.04149
1	10133.702	12071	5.672	8.03974	15.04223	1.34760	0.05364
1	10248.864	12696	6.092	6.02095	11.86842	1.37010	0.05353
2	10072.973	12909	6.349	6.18358	19.12447	1.36199	0.04692
2	10073.585	12809	6.439	4.13290	19.26218	1.38385	0.03512
3	10073.585	12809	6.312	6.64845	19.62612	1.30896	0.04192
3	10019.301	13268	6.826	4.35640	23.50929	1.49081	0.01357
3	10017.838	13266	6.793	5.19812	22.40548	1.52313	0.02132
3	10017.500	13265	6.752	6.20123	21.69495	1.46549	0.05511
4	10017.838	13266	7.044	2.50937	22.26818	1.45655	0.06420
5	10019.033	13267	6.863	4.47574	22.68375	1.50453	0.04195
7	10021.581	13484	7.081	3.57615	12.37598	1.37152	0.04986
2	10021.581	13484	6.975	4.89997	13.59221	1.37205	0.04961
6	10157.746	13893	7.361	2.90199	16.91375	1.47636	0.02545
1	10158.709	13881	7.189	3.44409	18.18760	1.42000	0.05004
2	10158.709	13881	7.189	4.57574	17.55217	1.42780	0.03586
4	10160.035	13883	7.120	5.31958	18.22939	1.41943	0.02855
1	10022.682	13994	7.514	3.54704	27.66859	1.55705	0.07743
2	10020.541	13991	7.497	3.61004	26.92461	1.52217	0.04342
3	10021.848	13992	7.383	5.13370	24.52623	1.59421	0.05023
4	10021.848	13993	7.383	4.62935	24.89686	1.53289	0.05824
5	10019.810	13990	8.081	3.41492	25.11366	1.56344	0.05265
6	10019.810	13990	8.081	1.23119	26.31236	1.59970	0.05007
7	10021.457	13992	7.380	5.07032	26.08453	1.59036	0.02760
8	10020.905	13991	7.575	2.69600	25.06758	1.61149	0.05042
9	10019.810	13990	7.627	2.47724	24.10710	1.61542	0.02522
1	10026.176	14138	7.973	3.80463	9.30459	1.28586	0.03403
2	10021.747	14256	7.692	4.24461	25.84377	1.49491	0.04801
3	10022.752	14260	7.592	4.93859	26.86664	1.64596	0.04900
4	10020.006	14257	7.984	4.45485	27.30308	1.54824	0.03671
5	10019.592	14256	8.499	2.19555	28.54738	1.51429	0.03346
6	10021.378	14259	7.933	0.88396	25.46452	1.60102	0.05848
7	10020.006	14256	7.692	3.74382	27.52673	1.54761	0.02850
8	10020.551	14257	7.888	2.27313	26.40368	1.65258	0.05447
9	10020.006	14256	7.726	3.66097	26.20502	1.59870	0.03252
10	10019.119	14255	7.712	3.32883	25.34196	1.58678	0.00899
11	10019.582	14256	7.727	3.11834	26.60151	1.50557	0.05511

Figure 5.3: STATISTICS FOR L-SERIES WITH $Q \ll 132$. PLOTTED BY MODULUS

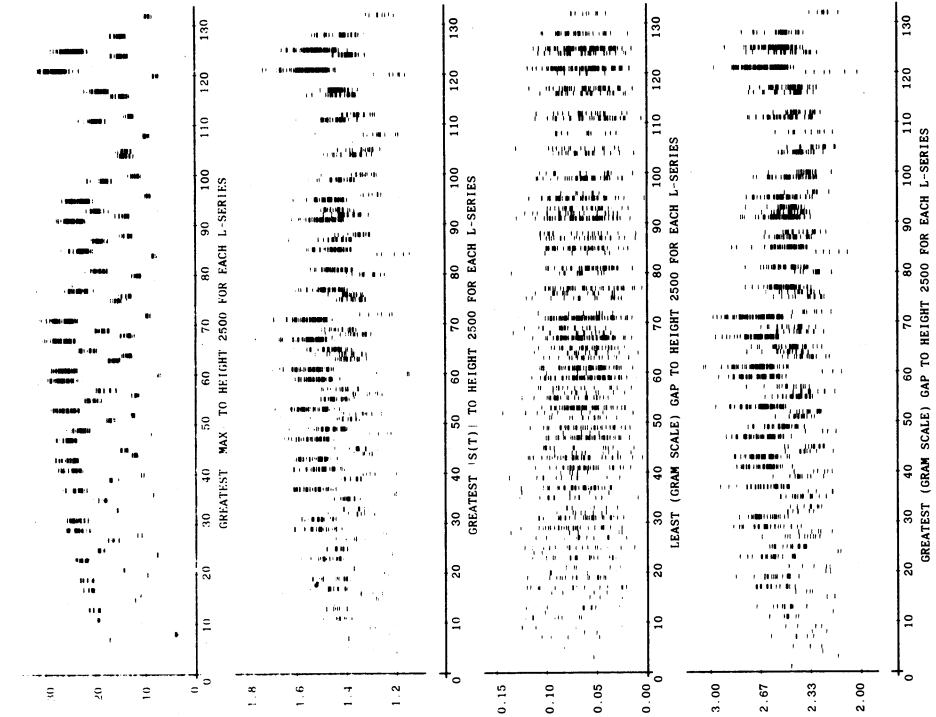
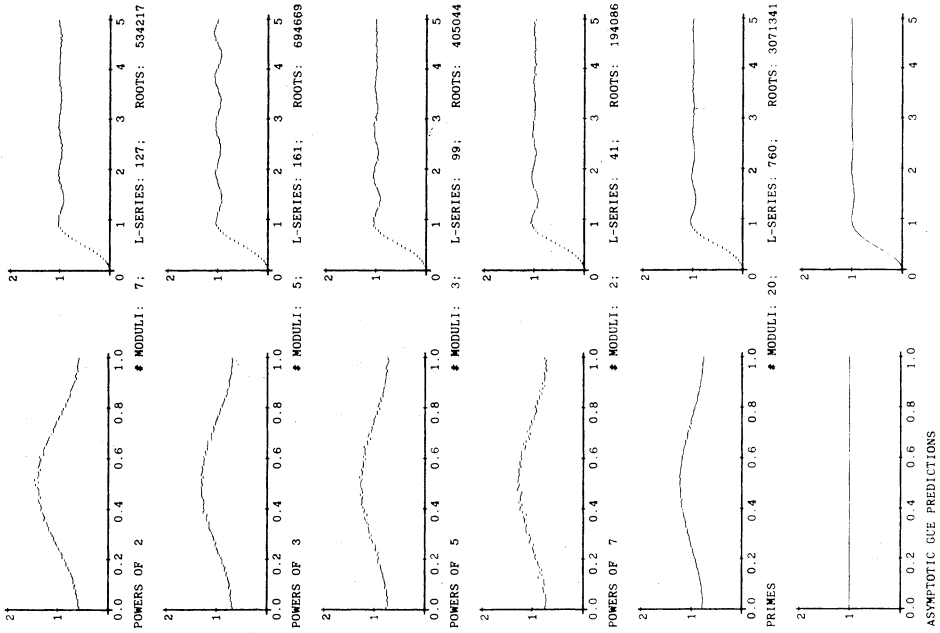


Table 5.5: ROOT COUNTS AND SUM $1/|R|$ BOUNDS FOR INDIVIDUAL L-SERIES, THROUGH MODULUS 13

K	K̄	100	500	1000	2500	5000	10000						
1	1	25	0.598	269	1.515	649	2.038	1985	2.845	4520	3.543	10142	4.318
Q = 1													
1	1	45	1.131	356	2.328	823	2.971	2421	3.938	5393	4.756	11891	5.654
Q = 3													
1	1	50	1.317	379	2.594	868	3.268	2536	4.278	5623	5.129	12349	6.057
Q = 4													
1	2	54	1.441	397	2.770	904	3.469	2624	4.510	5799	5.386	12704	6.339
2	1	53	1.518	397	2.859	904	3.558	2624	4.599	5801	5.476	12703	6.428
3	3	54	1.405	397	2.733	904	3.432	2624	4.473	5800	5.349	12703	6.302
Q = 5													
1	3	59	1.675	423	3.092	957	3.859	2758	4.920	6068	5.833	13239	6.823
2	4	59	1.643	423	3.060	958	3.798	2758	4.897	6068	5.801	13239	6.791
3	1	59	1.603	423	3.021	957	3.756	2758	4.846	6068	5.760	13239	6.750
4	2	59	1.890	424	3.312	958	4.048	2758	5.138	6069	6.052	13239	7.041
5	5	59	1.710	424	3.131	958	3.867	2759	4.957	6068	5.870	13239	6.860
Q = 6													
1	1	61	1.830	435	3.266	979	4.035	2811	5.145	6174	6.073	13453	7.078
2	2	61	1.725	434	3.179	979	3.930	2811	5.039	6174	5.968	13452	6.972
Q = 9													
1	3	63	2.003	444	3.488	998	4.252	2858	5.379	6268	6.320	13640	7.337
2	4	63	1.889	443	3.372	998	4.137	2858	5.264	6268	6.205	13639	7.222
3	1	63	1.832	443	3.314	997	4.079	2859	5.206	6268	6.147	13639	7.165
4	2	63	1.765	444	3.248	997	4.010	2858	5.137	6267	6.078	13639	7.096
Q = 11													
1	5	67	2.038	460	3.566	1029	4.351	2908	5.508	6428	6.471	13958	7.511
2	3	66	2.012	459	3.517	1029	4.334	2908	5.491	6428	6.454	13958	7.494
3	2	67	1.910	460	3.416	1028	4.220	2908	5.377	6427	6.340	13958	7.380
4	8	66	1.900	459	3.411	1030	4.221	2908	5.377	6427	6.340	13958	7.379
5	1	66	2.075	459	3.112	1029	4.399	2908	5.556	6428	6.519	13959	7.559
6	6	66	1.897	460	3.112	1030	4.319	2908	5.075	6428	6.038	13958	7.078
7	4	66	1.897	460	3.112	1030	4.319	2908	5.075	6428	6.038	13958	7.077
8	4	66	2.090	460	3.218	1030	4.418	2908	5.164	6427	6.532	13959	7.572
9	9	66	2.141	460	3.171	1030	4.465	2908	5.621	6428	6.584	13959	7.624
Q = 12													
1	1	67	2.021	466	3.581	1043	4.378	2973	5.547	6496	6.519	14097	7.569
Q = 13													
1	6	69	2.088	472	3.662	1048	4.468	3005	5.649	6561	6.631	14224	7.689
2	8	69	1.991	472	3.563	1048	4.369	3005	5.549	6561	6.531	14224	7.589
3	9	69	2.032	473	3.607	1047	4.410	3005	5.591	6560	6.573	14224	7.631
4	7	69	2.376	473	3.956	1047	4.781	3005	5.941	6561	6.923	14224	7.981
5	10	70	2.001	473	3.471	1047	4.276	3005	5.457	6561	6.438	14224	7.438
6	1	69	2.325	473	3.904	1047	4.708	3005	5.889	6561	6.871	14224	7.929
7	4	69	2.089	473	3.665	1048	4.469	3004	5.649	6560	6.631	14224	7.689
8	2	69	2.283	473	3.860	1048	4.664	3004	5.845	6561	6.827	14224	7.885
9	3	66	2.112	473	3.698	1048	4.503	3005	5.663	6561	6.665	14224	7.723
10	7	69	2.193	473	3.895	1048	4.589	3004	5.689	6560	6.651	14224	7.723
11	11	69	2.124	473	3.700	1048	4.504	3004	5.684	6560	6.666	14224	7.724

FIGURE 5.6: DISTRIBUTION OF ROOTS MODULO 1 AND PAIR CORRELATION FUNCTIONS



ASYMPTOTIC GUE PREDICTIONS

FIGURE 5.7: PAIR CORRELATION FUNCTIONS OF INDIVIDUAL L-SERIES

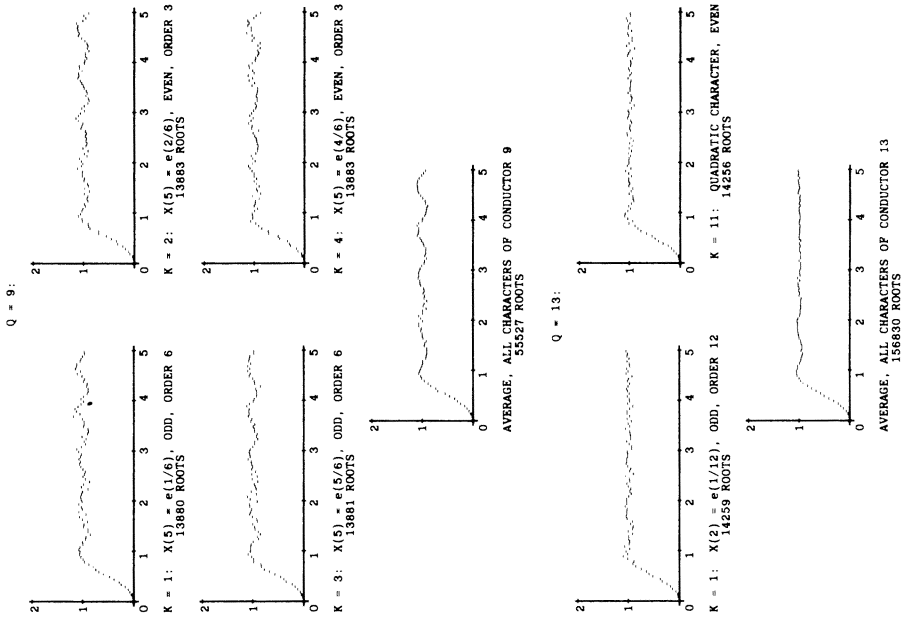


FIGURE 5.6 (Cont.): ROOTS MODULO 1 AND PAIR CORRELATION FUNCTIONS

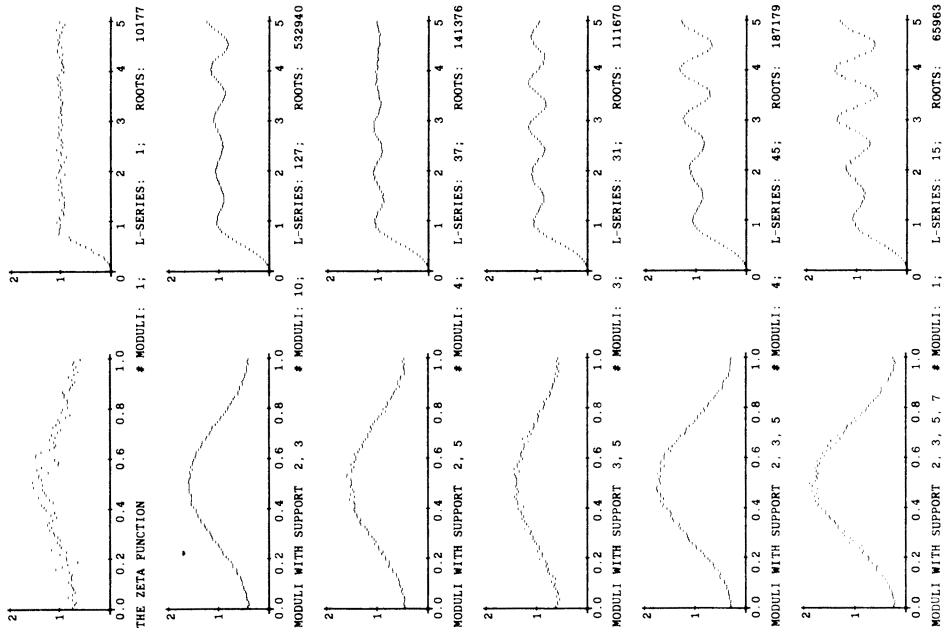


FIGURE 5.8: EXTENDED PLOTS OF PAIR CORRELATION FUNCTIONS

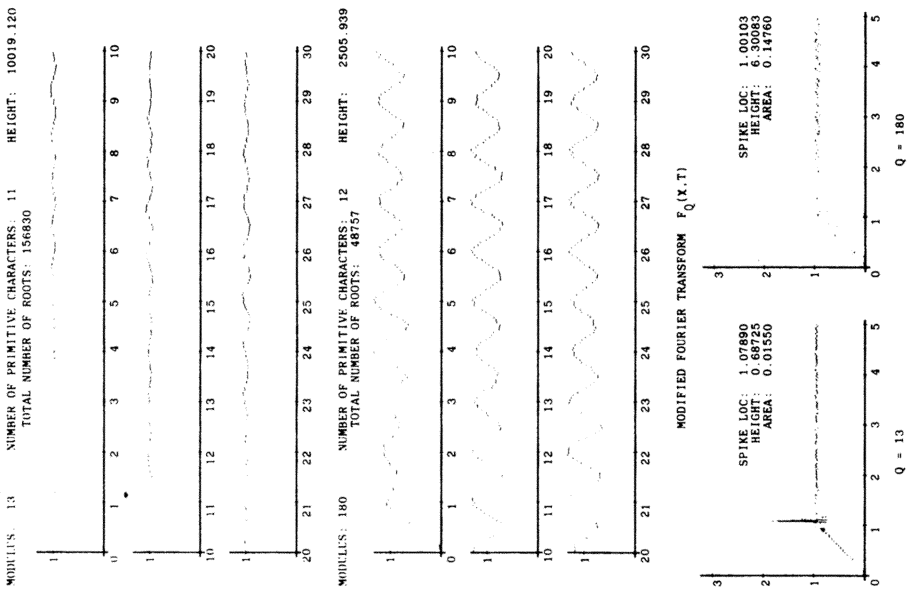


FIGURE 5.9: NEAREST NEIGHBOR AND SECOND NEAREST NEIGHBOR GAP DISTRIBUTIONS

NEAREST NEIGHBOR MOMENTS:

MOMENT TYPE	MULTIPLES OF 12	ALL MODULI	PRIME MODULI	ON GUE
2	0.138	0.149	0.158	0.180
3	0.011	0.022	0.030	0.038
4	0.052	0.067	0.079	0.101
5	0.014	0.031	0.045	0.066
6	0.031	0.055	0.074	0.111
7	0.016	0.046	0.072	0.124
8	0.026	0.068	0.105	0.197
9	0.019	0.078	0.132	0.290
10	0.027	0.112	0.194	0.488
LOG	-0.082	-0.086	-0.090	-0.104
1/7	1.218	1.208	1.248	1.248
1/7***2	2.185	2.247	2.254	2.563

SECOND NEAREST NEIGHBOR MOMENTS:

MOMENT TYPE	MULTIPLES OF 12	ALL MODULI	PRIME MODULI	ON GUE
2	0.140	0.181	0.206	0.249
3	-0.002	0.019	0.029	0.030
4	0.055	0.097	0.124	0.185
5	-0.003	0.031	0.050	0.073
6	0.034	0.087	0.123	0.237
7	-0.003	0.050	0.085	0.185
8	0.028	0.108	0.167	0.451
9	-0.004	0.090	0.158	0.544
10	0.028	0.169	0.280	1.178

SECOND NEAREST NEIGHBOR DENSITY PLOTS:

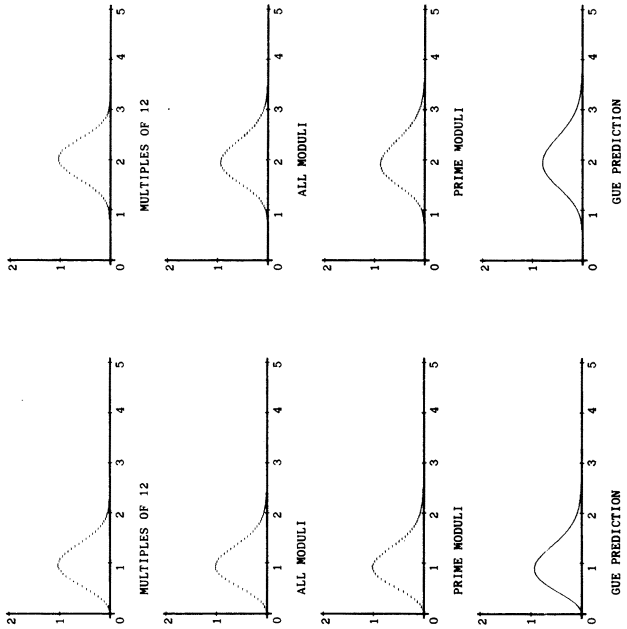


FIGURE 5.11: NEAREST NEIGHBOR GAP DISTRIBUTIONS FOR SEVERAL L-SERIES

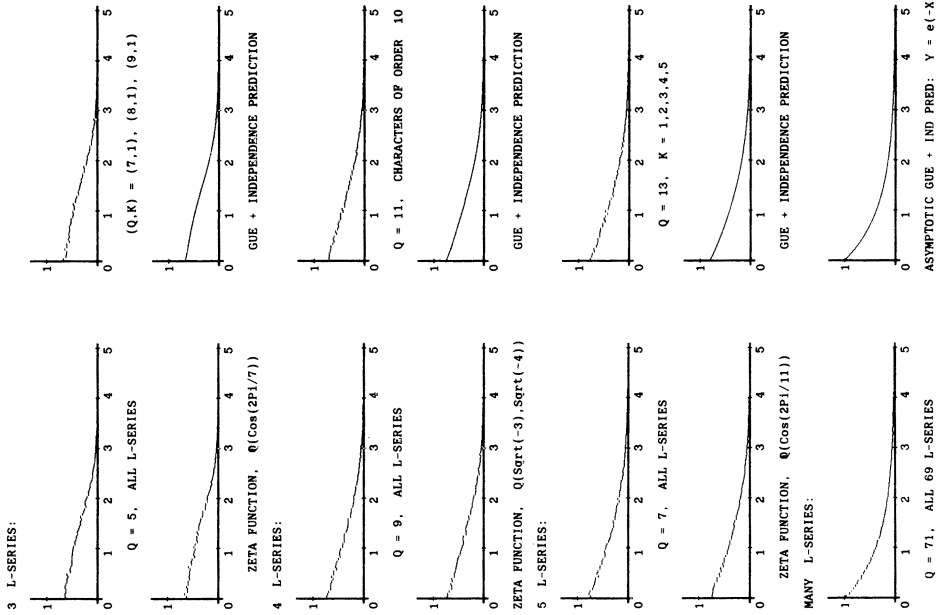


FIGURE 5.10: NEAREST NEIGHBOR GAP DISTRIBUTIONS FOR PAIRS OF L-SERIES

