

February 5, 2009
11:00 am–12:15 pm

MATH 2410(H)
EXAM #1

Dr. Sa'ar Hersonsky

Name: _____

1. (30 points) Calculate the following. No proofs are required, but give your reasoning.

a. $\int_0^4 (3x^2 + \sin(\pi x/4)) dx$

b. $\int_0^{\pi/2} \cos^7 x \sin x dx$

c. $\int_a^b (x + y) dx$

d. $\int_{-1}^1 x^3 \sqrt{1-x^2} dx$

e. $F'(x)$ where $F(x) = \int_0^x \cos^3\left(\frac{t}{1+t^8}\right) dt$

f. $G'(x)$ where $G(x) = \int_0^{\sqrt{x}} \cos^3\left(\frac{t}{1+t^8}\right) dt \quad (x > 0)$

2. (10 points) Find the area of the plane region bounded by the the graphs of $f(x) = x^2$ and $g(x) = 1 - x^2$ and $h(x) = 2$.

3. (15 points)
- a. Let f be a bounded function on $[a, b]$. Give a precise definition of the following statement. The function f is *integrable* on $[a, b]$.

b. Let

$$f(x) = \begin{cases} x + [x], & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Is f integrable on $[0, 2]$? (**justify your answer!**)

4. (15 points)

a. Suppose $f \geq 0$ and f is integrable on $[a, b]$. Prove that $\int_a^b f \geq 0$.

b. Suppose $f \leq g$ and f and g are integrable on $[a, b]$. Prove that $\int_a^b f \leq \int_a^b g$.

c. Suppose f and $|f|$ are integrable on $[a, b]$. Prove that $\left| \int_a^b f \right| \leq \int_a^b |f|$.

5. (20 points)

- a. Let f be integrable on $[a, b]$. Prove that for every $\epsilon > 0$ there is a partition P of $[a, b]$ such that

$$U(f, P) - L(f, P) < \epsilon.$$

- b. If f is continuous on $[a, b]$ and $f = g'$ for some function g , then

$$\int_a^b f = g(b) - g(a).$$

6. (10 points) Suppose f is integrable on $[0, 1]$ and $\int_0^1 f = 2$. Prove there is $x \in (0, 1)$ so that

$$\left(\int_0^x f\right)^2 = \int_x^1 f.$$

Good Luck!