

March 5, 2009
11:00 am–12:15 pm

MATH 2410(H)
EXAM #2

Dr. Sa'ar Hersensky

Name: _____

1. (15 points) Calculate the following. No proofs are required, but give your reasoning.

a. $\int \arctan x \, dx$

b. $\int \frac{1}{x \log x} \, dx$

c. $\int (\log x)^2 \, dx$

2. (15 points) Calculate the following. No proofs are required, but give your reasoning.

a. $\int \frac{a^x}{b^x} dx$

b. $\int \log(\cos x) \tan x dx$

c. $\int \frac{1}{x\sqrt{1-x^2}} dx$

3. (15 points) Differentiate each of the following functions.

a.

$$f(x) = \log(1 + \log(1 + \log(1 + \log(1 + e^{1+e^{1+x}}))))$$

b.

$$h(x) = (\sin x)^{(\sin x)^{\cos x}}$$

c.

$$g(x) = \sinh^{-1}(x)$$

4. (20 points)

a. Let $x > 0$. define $\log x$.

b. Using only the definition, prove that if $x, y > 0$, then

$$\log(xy) = \log x + \log y$$

.

c. Let x be any real number, define $\exp x$.

d. Using only the definition, prove that if x, y are any two numbers, then

$$\exp(x + y) = \exp x \exp y$$

.

5. (16 points)

a. Prove that if $f(x) = \int_0^x f(t) dt$, then $f = 0$.

b. Find all continuous functions f which satisfy the equation

$$(f(x))^2 = \int_0^x f(t) \frac{1}{1+t^2} dt$$

6. (15 points)

a. Prove that $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$.

b. Find a reduction formula for $\int x^n e^x dx$.

Good Luck!