

April 9, 2009  
11:00 am–12:15 pm

**MATH 2410(H)**  
**EXAM #3**

Dr. Sa'ar Hersensky

Name: \_\_\_\_\_

**Show and justify all of your work!**

1. (10 points) Find the length of the curve

$$y^2 = 4x^3,$$

between the points  $(0, 0)$  and  $(2, 4\sqrt{2})$ .

2. (10 points) Find the surface area of a sphere of radius  $a$ .

3. (20 points) Find the volume of the “torus”, obtained by rotating the circle

$$(x - a)^2 + y^2 = b^2 \quad (a > b),$$

around the vertical axis.

4. (20 points) A cylindrical hole of radius  $a$  is bored through the center of a sphere of radius  $2a$ . Find the volume of the remaining solid.

5. (20 points) Let  $f(x) = \begin{cases} \frac{e^x - 1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ .

a. Give the Taylor polynomial  $P_{n,0,f}$  of degree  $n$  at 0 of  $f$ . Prove that you are correct.

b. Use your answer to part a. to determine  $f^{(k)}(0)$ ,  $k = 0, 1, 2, \dots$ .

c. Give an explicit formula for the remainder.

6. (20 points)
- a. We wish to compute  $\sin 2$  with an error of less than  $10^{-4}$ . How many terms in Taylor polynomial of  $\sin x$  would it be sufficient to consider?
- b. Prove that  $\sin 1$  is not a rational number.

**Good Luck!**