

UNDECIDABILITY AND HIERARCHICAL APERIODIC TILINGS IN THE HYPERBOLIC PLANE

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ABSTRACT. For over forty years, the "Domino Problem" has been known to be undecidable in the Euclidean plane: that is, there is no algorithm to determine, in general, whether a given set of tiles can admit a tiling of the entire plane. That proof rests on constructing an aperiodic set of tiles that exhibits a kind of hierarchical structure, on which arbitrarily long runs of Turing machines can be implemented. Despite some attention, the problem has remained open in the hyperbolic plane— in particular, the Euclidean proof relies strongly on similarities, which are simply not available in this setting. Recently, however, M. Margenstern did find a way to adapt the Euclidean proof, giving a remarkable construction which we greatly simplified and present here.