

# GROMOV'S DIMENSION COMPARISON PROBLEM IN CARNOT GROUPS

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ABSTRACT. We find sharp comparison theorems relating Euclidean and Carnot-Carathéodory (CC) Hausdorff measures and dimensions on an arbitrary Carnot (nilpotent stratified Lie) group  $G$ . To show sharpness we construct sets of minimal CC dimension for fixed Euclidean dimension. Such sets are “horizontal”: they follow the lowest possible layers in the sub-Riemannian decomposition of the tangent bundle of  $G$ . Typically these sets are fractal from the perspective of both Euclidean and CC geometry. As a consequence, we obtain exact dimension formulas for a class of invariant sets of nonlinear, nonconformal Euclidean iterated function systems of polynomial type. Inspired by Falconer’s work on almost sure dimensions of Euclidean self-affine fractals we show that CC self-similar fractals are almost surely horizontal.