Fall, 2011

MATH 3500(H)
PROBLEM SET #11

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DUE Wednesday, November 2, 2011.

Problems to work but not hand in:

§4.3: #23.

§4.4: #4a, 5b,c,e, 8.

§4.5: #3, 6.

Problems to turn in:

WeBWork Homework 11

§4.3: #20 (3), 21 (4).

§4.4: #6 (3), 12 (4), 13 (4), 14 (2), 16 (4).

Challenge problems (Turn in separately):

A. (5) Consider the vector space ℓ^2 of all sequences $(x_1, x_2, x_3, \dots) \in \mathbb{R}^\omega$ that are square-summable, i.e., so that $\sum_{j=1}^{\infty} x_j^2$ converges.

(i) Check that this is actually a subspace of $\mathbb{R}^\omega$. The interesting part will be to argue that if $\mathbf{x}, \mathbf{y} \in \ell^2$, then $\mathbf{x} + \mathbf{y} \in \ell^2$.

(ii) This is actually what is called an inner product space; that is, there is a dot product, defined by $\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{j=1}^{\infty} x_j y_j$. Prove that this is well-defined and check briefly that it has the properties of dot product listed in Proposition 2.1 of Chapter 1.

(iii) Here's the interesting part. Check that $V = \{\mathbf{x} \in \ell^2 : \text{there exists } N \text{ so that } x_n = 0 \text{ for all } n \geq N\}$ is a subspace of ℓ^2. What is $V^\perp$? What is $V^{\perp\perp}$?

§4.4: #17 (3), 18 (5).