

Fall, 2011

MATH 3500(H)
PROBLEM SET #2

T. Shifrin

DUE Wednesday, August 31, 2011.

Problems to work but not hand in:

§1.3: #7, 8.

§1.4: #1b,f,g,h,k,l, 5b,e, 8, 12, 13, 17, 19, 23b,e,g,j,k, 24, 31.

Problems to turn in:

WeBWork Homework 2

§1.3: #9 (2), 10 (2).

§1.4: #2* (2), 4 (3), 10 (3), 14b,c (2), 15 (3), 25 (1), 30 (2).

A. (3) Let $\ell \subset \mathbb{R}^2$ be the line spanned by the nonzero vector $\mathbf{a} \in \mathbb{R}^2$ and define

$$P_\ell(\mathbf{x}) = \text{proj}_{\mathbf{a}}\mathbf{x}$$
$$R_\ell(\mathbf{x}) = P_\ell(\mathbf{x}) - \mathbf{x}^\perp,$$

where $\mathbf{x}^\perp = \mathbf{x} - P_\ell(\mathbf{x})$. Prove that P_ℓ and R_ℓ are linear maps.

B. (3) Suppose A is a square matrix satisfying the equation $A^3 - 2I = \mathbf{O}$. Prove that A and $A - I$ are both invertible. (**Hint:** Give explicit formulas for their inverses. In the case of the latter, think about some high school algebra to pick ? so that $(A - I)(?) = I$.)

Challenge problems (Turn in separately):

§1.3: #11[†] (3), 12 (3).

§1.4: #37 (3).

*Hint: What earlier homework problem is relevant?

[†]This particular problem must be turned in within a few weeks.