

MATH 3500(H)
PROBLEM SET #7

DUE Wednesday, October 5, 2011.

Problems to work but not hand in:

§3.3: #1, 2, 5.

§3.4: #1a, 2b, 3.

Problems to turn in:

WeBWork Homework 7 (See the footnote below.)

§3.2: #15 (3), 18 (3).

§3.3: #11* (3), 12 (3), 15 (3).

A. (3) Suppose $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ is differentiable and we define $F \begin{pmatrix} u \\ v \end{pmatrix} = f \begin{pmatrix} e^u \cos v \\ e^u \sin v \end{pmatrix}$. Use the chain rule carefully to compute $\left(\frac{\partial F}{\partial u}\right)^2 + \left(\frac{\partial F}{\partial v}\right)^2$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, evaluated at the appropriate spot.

§3.4: #5 (3), 6 (2).

B. (3) Suppose $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ is differentiable and $\frac{\partial f}{\partial x} + 2x\frac{\partial f}{\partial y} = 0$ everywhere. Find the level curves of f . (Hint: From knowing the direction of the gradient, you should figure out the slope of the level curve through $\begin{bmatrix} x \\ y \end{bmatrix}$ and solve the differential equation $dy/dx = \dots$) Deduce that if we know f on the y -axis, then we know f everywhere in the plane: In particular, if $f \begin{pmatrix} 0 \\ y \end{pmatrix} = F(y)$, give a formula for f .

Challenge problems (Turn in separately):

—SEE OVER—

*Please add the hypothesis that f is differentiable. Also, contemplate to which WeBWork problems this result might be relevant.

#3.1.13 and #3.2.13 (3).

§3.2: #19 (5).

C. (3) Show that for any $\mathbf{a} \neq \mathbf{0}$ we have

$$\lim_{\mathbf{h} \rightarrow \mathbf{0}} \frac{\|\mathbf{a} + \mathbf{h}\| - \|\mathbf{a}\| - \frac{\mathbf{a} \cdot \mathbf{h}}{\|\mathbf{a}\|}}{\|\mathbf{h}\|} = 0.$$

(Hint: Remember that $x - y = \frac{x^2 - y^2}{x + y}$. This is the same “conjugate trick” you used numerous times in single-variable calculus.) If $f(\mathbf{x}) = \|\mathbf{x}\|$ and $\mathbf{a} \neq \mathbf{0}$, why is f differentiable at \mathbf{a} and what is $Df(\mathbf{a})$?

D. (4) Give a criterion (in terms of α , β , γ , and δ) for the function in §2.3, #17 to be differentiable at $\mathbf{0}$.

E. (5) A basketball is thrown with initial speed v at initial angle θ from the origin and it is to pass through the hoop at $\begin{bmatrix} a \\ b \end{bmatrix}$. For present purposes, assume a point ball and ignore spin.

- (i) What is the optimal angle to shoot? (For control, it is reasonable that you shoot with minimum speed.)
- (ii) Show that in some reasonable sense this also gives you optimal “margin for error”: Fixing b , say, with small changes in v and θ , you get small(est?) change in a .
- (iii) Of course, the real-life solution will depend on the height of the free-throw shooter. But here are some facts: The rim is 10 ft. high and the the free-throw line is 15 ft. horizontally from the basket. What are the ideal v and θ for someone 6 ft. tall?

§3.4: #7 (3), 8 (3), 9 (3), 13 (4), 15 (4).