Sample Test Problems for Chapter 5, Sections 5.5 – 5.7

1. Write a "how many groups?" story problem for $\frac{2}{3} \div \frac{1}{2}$ and solve your problem in a simple and concrete way without using the "invert and multiply" procedure. Explain your reasoning.

2. Write a "how many groups?" story problem for $2\frac{1}{2} \div \frac{3}{4}$ and solve your problem in a simple and concrete way without using the "invert and multiply" procedure. Explain your reasoning.

3. Write a "how many in one group?" story problem for $\frac{1}{2} \div \frac{1}{3}$ and use your story problem to explain why it makes sense to solve $\frac{1}{2} \div \frac{1}{3}$ by "inverting and multiplying," in other words, by multiplying $\frac{1}{2}$ by $\frac{3}{1}$.

4. Write a "how many in one group?" story problem for $6 \div \frac{3}{4}$ and use your story problem to explain why it makes sense to solve $6 \div \frac{3}{4}$ by "inverting and multiplying," in other words, by multiplying 6 by $\frac{4}{3}$.

5. Write a story problem (any type) for $\frac{1}{2} \div \frac{2}{3}$ and solve your problem in a simple and concrete way without using the "invert and multiply" procedure. Explain your reasoning.

6. Sally is working on the division problem $1\frac{3}{4} \div \frac{1}{2}$ by using the story problem, "how many $\frac{1}{2}$ cups of water are in $1\frac{3}{4}$ cups of water?" Sally draws a picture like the one in Figure 1 and gives the answer $3\frac{1}{4}$. When Sally checks her work by "inverting and multiplying," she realizes that she has made a mistake. Her answer should be $3\frac{1}{2}$, not $3\frac{1}{4}$. How should Sally interpret her picture so as to get the correct answer, $3\frac{1}{2}$?

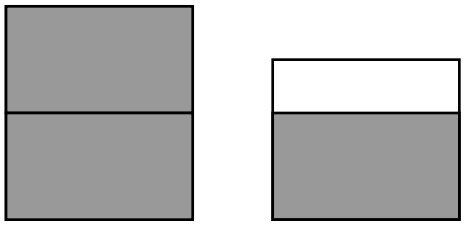


Figure 1: A Picture for $1\frac{3}{4} \div \frac{1}{2}$

7. Which of the following are story problems for the division problem $\frac{2}{3} \div \frac{1}{2}$? For those that are, which interpretation of division is used? For those that cannot be solved by $\frac{2}{3} \div \frac{1}{2}$, determine how to solve the problem another way, if it can be solved.

   (a) $\frac{2}{3}$ of a bag of jelly worms make $\frac{1}{2}$ a cup. How many bags of jelly worms does it take to make one cup?

   (b) $\frac{2}{3}$ of a bag of jelly worms make $\frac{1}{2}$ a cup. How many cups of jelly worms are in one bag?

(c) You have $\frac{2}{3}$ of a container of chocolate syrup and a recipe that calls for $\frac{1}{2}$ of a cup of chocolate syrup. How many batches of your recipe can you make (assuming you have enough of the other ingredients)?

(d) You have $\frac{2}{3}$ of a cup of chocolate syrup and a recipe that calls for $\frac{1}{2}$ of a cup of chocolate syrup. How many batches of your recipe can you make (assuming you have enough of the other ingredients)?

(e) If $\frac{2}{3}$ of a pound of nails costs $\frac{1}{2}$ of a dollar, then how many pounds of nails should you be able to buy for 1 dollar?

(f) If you have $\frac{2}{3}$ of a pound of nails and you divide the nails in $\frac{1}{2}$, then how many pounds of nails will you have in each portion?

(g) If $\frac{1}{2}$ of a pound of nails costs \$1, then how much should you expect to pay for $\frac{2}{3}$ of a pound of nails?

8. Fraction division story problems involve the simultaneous use of different wholes. Solve the following recipe problem in a simple and concrete way without using the "invert and multiply" procedure. Describe how you must work simultaneously with different wholes in solving the problem.

   Recipe problem: A recipe calls for $\frac{2}{3}$ of a cup of tomato juice. You have $2\frac{1}{3}$ cups of tomato juice. Assuming that you have enough of the other ingredients, and that you can make fractional batches, how many batches of your recipe can you make?

9. Discuss the difference between dividing *in* half and dividing *by* $\frac{1}{2}$. Your discussion should include:

   (a) a description of ways to express these two situations using mathematical notation,

   (b) story problems for both situations; say which is which and explain how to solve the problems using simple, concrete reasoning.

10. Write one story problem for $\frac{3}{4} \div \frac{1}{2}$ and another story problem for $\frac{3}{4} \times \frac{1}{2}$, making clear which problem is which. In each case, explain how to use simple, concrete reasoning to solve your problem.

11. For each of the following problems, write a corresponding division problem, say which interpretation of division is involved, and solve the problem without using a calculator.

    (a) Marjorie needs $\frac{3}{4}$ of a pound of yarn to knit a scarf that is $6\frac{1}{2}$ feet long. How long a scarf can Marjorie knit with 1 pound of yarn?

    (b) Marjorie needs $\frac{3}{4}$ of a pound of yarn to knit a scarf that is $6\frac{1}{2}$ feet long. How many many pounds of yarn will Marjorie need for a 1 foot long section of scarf?

    (c) Marjorie needs $\frac{3}{4}$ of a pound of yarn to knit a scarf that is $6\frac{1}{2}$ feet long. How many scarfs (each of which is $6\frac{1}{2}$ feet long) can Marjorie knit with 10 pounds of yarn?

12. Solve the following problems in any way you like as long as you don't use a calculator. Show your work and explain your reasoning briefly.

(a) Liquid pours out of a hose and into a vat at a steady rate. Starting with an empty vat, it took $2\frac{1}{2}$ hours to fill the vat $\frac{3}{4}$ full. How long will it take to fill the vat completely?

(b) Liquid pours out of a hose and into a vat at a steady rate. Starting with an empty vat, it took $2\frac{1}{2}$ hours to fill the vat $\frac{3}{4}$ full. What fraction of the vat was filled after 1 hour?

(c) Liz needs $3\frac{1}{2}$ bags of cement mix to make a walkway but she only has $2\frac{3}{4}$ bags. What fraction of the walkway can she make with the cement mix she has?

13. Give an example of either a hands-on activity or a story problem for elementary school children that is related to a fraction division problem (even if the children wouldn't think of the activity or problem as fraction division). Write the fraction division problem that is related to your activity or story problem. Describe how the children could solve the problem by using logical thinking aided by physical actions or by drawing pictures.

14. Ed says that $4 \div 5$ doesn't make sense because you can't divide a smaller number by a bigger number. Give Ed two sensible story problems for $4 \div 5$, one for each of the two interpretations of division.

15. Seyong says that $2 \div \frac{1}{3}$ can't be equal to 6 because 6 is greater than 2 but when you divide a number, the answer must be smaller than the number you started with. Write a story problem for $2 \div \frac{1}{3}$ and use simple, concrete reasoning to help you explain why $2 \div \frac{1}{3}$ really is equal to 6. (Do not just "invert and multiply" to explain why $2 \div \frac{1}{3} = 6$.)

16. In ordinary language, the term "divide" means "partition and make smaller," as in:

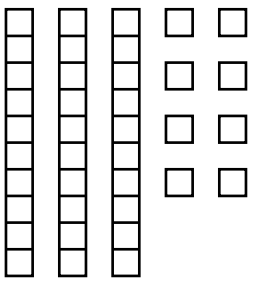
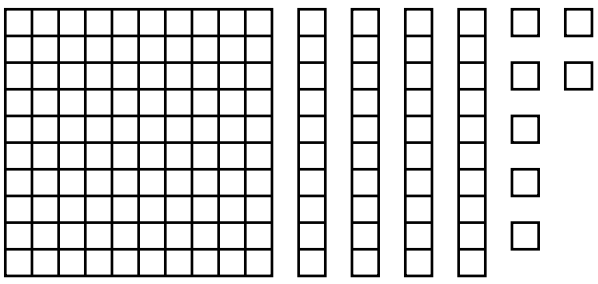
    Divide and conquer.

    In mathematics, does dividing always make smaller? In other words, if you start with a number N and divide it by another number M, is the resulting quotient $N \div M$ necessarily less than N?

17. Is it correct to characterize division as "breaking down" and "making smaller"? If not, why not?

18. Show how to calculate $1.5 \div 0.0004$ without a calculator.

19. Which of the following division problems can be interpreted as asking:

How manygroupsof  are in  ?

$14.7 \div 3.8 =?$ $1.47 \div 3.8 =?$ $1.47 \div .38 =?$ $147 \div 38 =?$

$$3.8 \div 14.7 = ? \quad 3.8 \div 1.47 = ? \quad .38 \div 1.47 = ? \quad 38 \div 147 = ?$$

Circle all that apply and explain your reasoning.

20. (a) Calculate $2.14 \div 0.7$ to the hundredths place without a calculator. Show your work.
    (b) Describe the standard procedure for determining where to put the decimal point in the answer to $2.14 \div 0.7$.
    (c) Explain in two different ways why the placement of the decimal point that you described in part (b) is valid.

21. (a) Calculate $7.3 \div 0.21$ to the hundredths place without a calculator. Show your work.
    (b) Describe the standard procedure for determining where to put the decimal point in the answer to $7.3 \div 0.21$.
    (c) Explain in two different ways why the placement of the decimal point that you described in part (b) is valid.

22. Bruce must calculate $7.82 \div 1.6$ longhand, but he can't remember what to do about decimal points. Instead, Bruce solves the division problem $782 \div 16$ longhand and gets the answer 48.875. Bruce knows that he must shift the decimal point in 48.875 somehow to get the correct answer to $7.82 \div 1.6$. Explain how Bruce could reason about the sizes of the numbers to determine where to put the decimal point.

23. Without calculating the answers, explain why the division problems

$$13,000,000,000 \div 8,000,000,000$$

and

$$13 \div 8$$

are equivalent.

24. If the federal budget is \$2.2 trillion and this budget were paid for equally by the 290 million residents of the U.S., then how much would each person have to contribute to the federal budget? Assume you only have a very simple calculator that cannot use scientific notation and that displays at most 8 digits. Describe how to use such a calculator to solve the problem about the federal budget and explain why your solution method is valid.

25. If 2.6 liters of a liquid weigh 3.1 kilograms, then how many liters of the liquid weigh 1 kilogram? How much does 1 liter of the liquid weigh? Explain why you can use division to solve both of these problems. Solve the problems.

26. If cabbage costs \$0.23 per pound then how many pounds of cabbage can you buy for \$3.50? Explain why you can use division to solve this problem. Solve the problem.

27. Write a "how many groups?" story problem for $3.8 \div 1.4$.

28. Write a "how many in one group?" story problem for $3.8 \div 1.4$.

29. Write a story problem (any type) for $0.85 \div 3.5$.

30. Susie mixed 2 cups of yellow paint with 1 cup of red paint to make an orange paint. How many cups of yellow paint and how many cups of red paint will Suzy need to make 12 cups of the same shade of orange paint? Solve this problem and explain your solution in a way that 3rd, 4th or 5th graders could understand.

31. Bob mixed 3 cups of yellow paint with 1 cup of blue paint to make a green paint. How many cups of yellow paint and how many cups of blue paint will Bob need to make 10 cups of the same shade of green paint? Solve this problem and explain your solution in a way that 4th or 5th graders could understand.

32. Maya mixed 2 cups of yellow paint with 3 cup of red paint to make an orange paint. For each of the following fractions and division problems, interpret the fraction or the division problem in terms of Susie's paint mixture and explain why your interpretation makes sense.

    $\frac{2}{5}$ or $2 \div 5$; $\frac{2}{3}$ or $2 \div 3$; $\frac{3}{2}$ or $3 \div 2$; $\frac{5}{2}$ or $5 \div 2$; $\frac{5}{3}$ or $5 \div 3$.

33. Which of the following two mixtures will be more salty?

    - 2 tablespoons of salt mixed in 7 cups of water or
    - 3 tablespoons of salt mixed in 8 cups of water

    Solve this problem in two different ways, explaining both solutions in detail.

34. Carmina mixed $\frac{1}{4}$ cup of strawberry frosting with $\frac{1}{3}$ of a cup of lemon frosting and thought that the mixture was just right. Carmina needs 2 cups of her frosting mixture. How many cups of strawberry frosting and how many cups of lemon frosting will Carmina need? Solve this problem using only multiplication and division together with logical reasoning. Explain your reasoning.

35. Sam mixed 3 cups of blue paint with 4 cups of red paint to make a purple paint. Then Sam put another 2 cups of red paint in his mixture. How many cups of blue paint should Sam add to return his paint to its original shade of purple?

    (a) Solve this problem and explain your solution in a way that 4th or 5th graders could understand.

    (b) Solve this problem by setting up a proportion in which you set two fractions equal to each other.

    (c) Interpret the two fractions that you set equal to each other in part (b) in terms of the paint and explain why it makes sense to set these two fractions equal to each other.

36. You need $1\frac{1}{2}$ cups of grape juice for a recipe that makes 4 servings. How many cups of grape juice will you need if you want to make 10 servings of the same recipe?

    (a) Solve this problem by setting up a proportion in which you set two fractions equal to each other.

    (b) Interpret the two fractions that you set equal to each other in part (a) in terms of the recipe. Explain why it makes sense to set these two fractions equal to each other.

(c) Why does it make sense to cross-multiply the two fractions in part (a)? What is the logic behind the procedure of cross-multiplying?

37. To make a shade of orange paint that you like, you must mix $\frac{2}{3}$ of a bottle of red paint with each $\frac{4}{5}$ of a bottle of yellow paint that you use. You need 88 bottles of this orange paint. How many bottles of red paint will you need and how many bottles of yellow paint will you need? (All bottles are the same size.)

38. A factory makes an oil mixture by mixing oils of grades A and B as follows. For every 2.3 liters of grade A oil, 1.2 liters of grade B oil are mixed in. How many liters of grade A oil and grade B oil will the factory need to make 1000 liters of their oil mixture?

39. Alice and Jose are planning to mix red and yellow paint. They are considering which of the two following paint mixtures will make a more yellow paint:

    - a mixture of 2 parts red to 7 parts yellow
    - a mixture of 3 parts red to 8 parts yellow

    Alice says that both paints will look the same because to make the second mixture you just add one part of each color to the first mixture. Because you add the same amount of each color, the second mixture should look the same as the first mixture. Jose says that the second mixture should be more yellow than the first because it uses 8 parts yellow and the first mixture only uses 7 parts yellow.

    (a) Discuss the children's ideas. Is their reasoning valid or not?

    (b) Which paint will be more yellow and why?

40. If 3 bulldozers can load 5 trucks full of earth in 30 minutes, then how long should it take 5 bulldozers to load 7 trucks full of earth? Assume that all the bulldozers work equally hard and all truck loads are the same size. Solve the problem, explaining your reasoning.