

## Advice and Recommended Teaching Practices MATH 5001, Arithmetic and Problem Solving

by Sybilla Beckmann  
December, 2000

One of the challenges in teaching this course is that one must convince students that doing mathematics is more than just applying learned algorithms, and that teachers should know their subject at much greater depth than they will teach. I have found that many or most students enter the course thinking we will discuss how to teach math to elementary school children. The students are sometimes skeptical of the depth we go into, and make comments like “elementary school kids would never understand this.” I recommend that you treat such comments (or similar questions) with respect, and address them very seriously. Most of these students have not yet had much, if any, experience with teaching, nor have they had many education courses (if any). Since MATH 5001 is required *aaaaa* students can enter the Early Childhood Education program, these students are at the very early stages of their training to become teachers (in fact, since the program is so competitive, many of the students in the class will not be accepted into this program). I tell the students that the goal of the course is for them to learn the mathematics taught in elementary school in depth, and that there is research literature indicating that this depth of understanding is needed for effective teaching (some of this literature is quoted in the book). The problems the students work on are meant to be problems for them, not problems to give to their students (although many could be adapted to become problems for their students). The course is not meant to discuss what to do with kids—this will be discussed in their methods courses.

I urge you to take great care never to be condescending to students or to the course material. By and large, prospective elementary teachers are not particularly drawn to mathematics (although some are, and a number are quite able), and therefore it is easy for them to feel intimidated by a math professor. I think it pays to be as kind and gentle as possible while offering genuine, constructive criticism. I do think this is fully compatible with holding students to high standards and challenging them. We *a ust* criticize, but we can try to do this in a way that is helpful and that is not condescending or rude.

The course is centered on “explaining why.” Students should discuss and generate explanations, both orally, and in writing. The goal should be for students to write explanations that would convince a skeptic who is not in the course. I think that it is more important for students to be able to give clear “explanatory arguments” as opposed to fully rigorous, complete proofs. Bear in mind that the course is meant to be part of a program designed to train students for effective teaching of mathematics at the elementary level. This colors the presentation of the material. For example, whereas a math major might be expected to accept the standard procedure for fraction multiplication as a definition, and to recognize that this definition makes sense because it agrees with the definition for the integers and turns the rational numbers into a field, a different treatment of fraction multiplication will be more useful for prospective elementary teachers. These teachers should be able to give examples of story problems for a given fraction multiplication problem, and they should be able to explain, in the context of a story, why the answer provided by the standard fraction multiplication procedure agrees with what one expects. Thus the focus is more on sense-making than on a rigorous development.

In addition, teachers should be able to analyze misconceptions and to explain where the problem lies. For example, if one draws a picture like the one shown below, it would be easy to conclude that  $\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$ . Analyzing



mistakes or misconceptions often provides students opportunities to confront fundamental issues; in this case, the issue of the underlying “whole.”

Another difference between the explanations in this course and in more advanced math courses is that here we usually aim for several different kinds of explanations. Once again, there is a practical motive: teachers must be trained to bring diverse students to high levels of learning. A teacher who only knows one way to think about the material will not be well prepared for this difficult task.

In order for students to practice written explanations, homework should be an important part of the course. I recommend that you ask students to write clear, complete sentences (which can, of course, include mathematical

symbols). Pictures and diagrams should be encouraged wherever they are helpful. You might want to encourage students to work together on their homework, with the proviso that they write it up on their own.

Most students are used to “doing ten problems just like the teacher showed us.” Some students feel quite certain that this is what math teaching should be like; they may need to be convinced that they *aaa* think their way through problems that are not exactly like ones they’ve seen before. Students should be reminded periodically to read problems carefully and to think critically about their explanations, since these are common stumbling blocks. Most students are simply not used to writing detailed explanations in a math course, and the adjustment often takes some time. Encourage students to have a positive “can do” attitude and to persist in trying to make sense of mathematics. I have had students who struggled and struggled but finally, near the end of the course, started to catch on and do much better.

The text includes enough “class activities” so that you can center your classes on these and do only minimal lecturing. Typically, I briefly introduce the topic or issues the students will be thinking about in the class activity, and then ask the students to work together in small groups. While the students work together, I circulate around the room, see what they are doing, and help them out if they get stuck. Some groups are efficient and hard working; others tend to stray from the task. I don’t have any solutions to offer for the latter situation, but since these students are adults, I put the responsibility for learning on them. The class provides the opportunity. You might want to tell the students periodically that they will learn the material much better if they engage themselves in it and first try to figure it out on their own rather than have someone else explain it to them. Therefore quicker students should be reminded to “hold back” a little in order to give their classmates time to think as well. You might also tell students that they will only be able to foster behavior in their students that they foster in themselves. If they want their students to be active and engaged, then they will need to expect that of themselves as well.

I strongly recommend that after the students have worked on a class activity, you have a class discussion about the material, pulling it together and clearing up any questions or misconceptions.