Sample Problems for MATH 5002, University of Georgia

1. The picture in Figure 1 shows the Earth and Moon as seen from outer space, looking down on the North Pole (not to scale!).

   (a) For a person located at point P, approximately what time is it? Use commonly known facts to help you explain your answer.

   (b) Is the moon waxing or waning in Figure 1? Explain why.

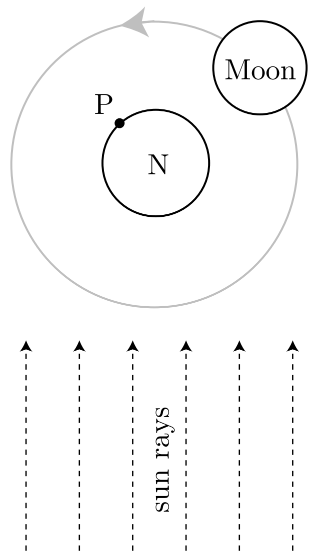


Figure 1: The Earth and Moon

2. Discuss the two different ways of defining the concept of *angle*. Describe how to use an "angle explorer," shown in Figure 2, to relate the two definitions.

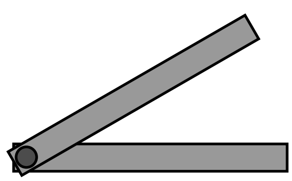


Figure 2: An "Angle Explorer"

3. Town B is 50 miles due east of Town A, as shown on the map in Figure 3. Town C is 30 miles from Town A and 40 miles from Town B. Explain why the *mathematical definition of circle* is relevant to determining where Town C could be located on the map.

4. The line segments AB and AC in Figure 4 have been constructed so that they could be two sides of a rhombus.

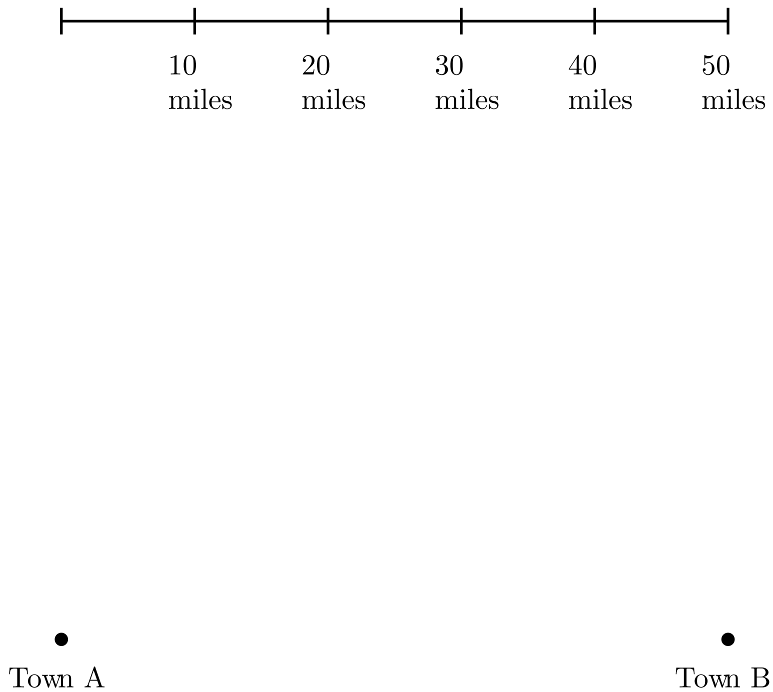


Figure 3: Two Towns on a Map

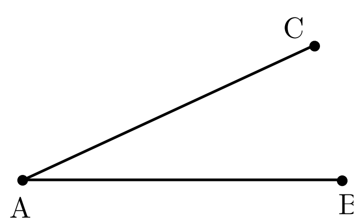


Figure 4: The Beginning of a Construction of a Rhombus

(a) Use a compass and straightedge to finish constructing a rhombus that has AB and AC as two of its sides.

(b) By referring to the definition of rhombus, explain why your construction must produce a rhombus.

5. (a) Use a compass and straightedge to construct a line that is perpendicular to the line segment AB in Figure 5 and that divides the line segment AB in half.

Figure 5: A Line Segment

(b) Explain clearly how and why the construction you just did in part (a) is related to special properties of rhombuses.

6. (a) Draw a Venn diagram showing the relationships between the sets of squares, rectangles, and parallelograms.

(b) Discuss whether the relationships in part (a) can be determined immediately from the definitions of these shapes or whether additional information that derives from the definitions but is not stated directly in the definitions is necessary. In other words, if a blind person knew only the definitions of the shapes, which relationships would they be able to deduce immediately and which would they not?

7. Suppose that builders pound four stakes into the ground to mark the four corners of a foundation for a house. The builders then measure the distances between opposite stakes and find that these distances are not the same. If the builders proceed to lay the house's foundation without moving the stakes, what can you conclude about this foundation? Relate this question to properties of shapes.

8. For each of the patterns in Figure 6, name the shape it would make if it were cut out, folded, and taped to make a closed shape. (Do this without cutting and folding!) In each case, label the base(s) (if any). Determine whether the first shape is oblique or right.

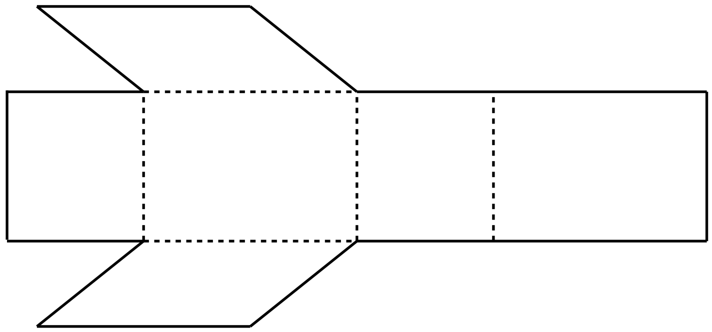
9. Answer the following without the use of a model.

(a) How many faces does a prism with a pentagon base have? What shapes are the faces? Explain briefly.

(b) How many edges does a prism with a pentagon base have? Explain.

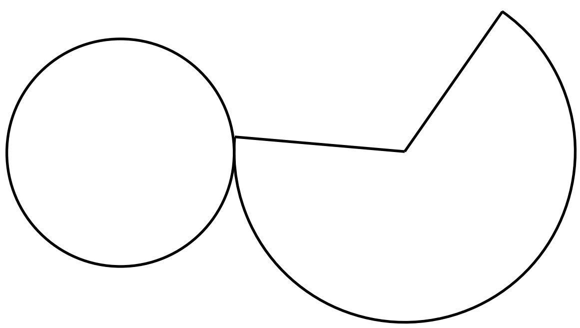
(c) How many corners does a prism with a pentagon base have? Explain.

10. Draw two (fairly) precise patterns: one for a triangular prism and one for a pyramid with a triangle base. Indicate which is which. Indicate which sides would be joined together if you were to fold up the patterns to form the shapes.

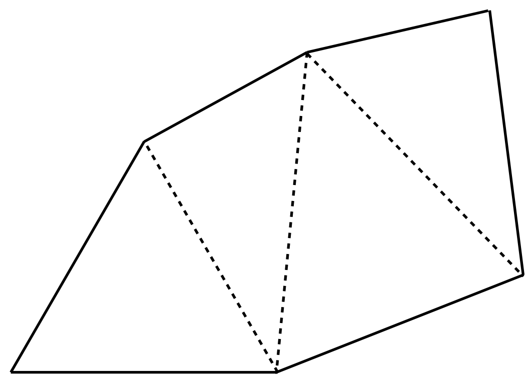


precise name of shape:
label the base(s)

oblique or not?



precise name of shape:
label the base(s)



precise name of shape:
label the base(s)

Figure 6: Patterns

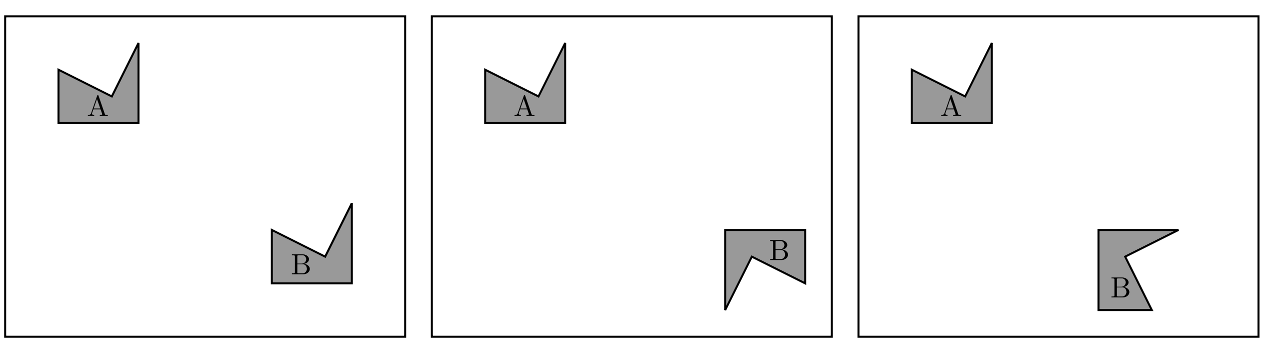


Figure 7: Three Pictures

11. For each of the three pictures in Figure 7, describe a *single* transformation that takes shape A to shape B. Draw marks on the pictures to help you describe the transformations as precisely as possible.

12. Draw a design that is made with copies of the curlicue in Figure 8 (and its reflection) so that the *design as a whole* has both 3-fold rotation symmetry *and* reflection symmetry. (Artistry is not required—a rough sketch will do as long as it shows the desired features clearly.)

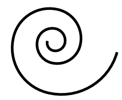


Figure 8: A Curlicue

13. Ada, Bada and Cada are three cities. Bada is 20 miles from Ada, Cada is 30 miles from Bada, and Ada is 40 miles from Cada. There are straight line roads between Ada and Bada, Ada and Cada, and Bada and Cada.

    (a) Draw a careful and precise map showing Ada, Bada, and Cada and the roads between them, using a scale of 10 miles = 1 inch. Describe how to use a compass to make a precise drawing.

    (b) If you were to draw another map, or if you were to compare your map to a classmate's, how would they compare? In what ways might the maps differ, in what ways would they be the same? Which criterion for triangle congruence is most relevant to these questions?

14. Suppose you make a triangle by threading three pieces of straw onto a string and tying the ends of the string together to make a loop. Similarly, suppose you make a quadrilateral by threading four pieces of straw onto a string and tying a loop. Describe the structural difference between the triangle and the quadrilateral (*other than* the fact that the triangle is made out of three pieces of straw and the quadrilateral is made out of four), and explain how this structural difference is related to the concept of congruence.

15. A problem about enlarging a poster: A poster with a simple picture is 2 feet wide and 4 feet tall. You want to make a larger version of the poster on a poster that is 6 feet wide. How tall should the poster be?

    Describe two different simple ways that you could solve the poster problem. Both ways should be understandable to 4th or 5th graders who know about multiplication and division (but do not know about setting up proportions). In each case, explain clearly why your method of solution makes sense.

16. Ms. Nice's 5th grade class wants to figure out how tall the school flagpole is. On a sunny day, the class goes outside and measures that the shadow of the flagpole is about 21 feet long. At the same time, Juan's shadow is 3 feet, 4 inches long. Juan is 4 feet, 3 inches tall.

(a) Determine the (approximate) height of the flagpole using a method that the children in Ms. Nice's class could find plausible. The children know about multiplication and division, but they don't know any more advanced mathematics, such as setting up proportions. The class is allowed to use calculators when multiplying and dividing decimals or fractions.

(b) Now add details to your explanation in part (a) in order to make your explanation more thorough for someone who has a more advanced knowledge of mathematics. Draw a picture showing that the flagpole problem involves similar triangles. Explain why the triangles must be similar.

17. Jenny wants to know what it means when we say that a tank is 284 cubic feet. What can you tell Jenny?

18. What is special about the way units in the metric system are named? Give several examples to illustrate.

19. What is the difference between reporting that something weighs 2 kilograms and that it weighs 2.0 kilograms?

20. Roger is calculating the distance from town A to town C. Roger is given that the distance from town A to town B is 240 miles, the distance from town B to town C is 350 miles, that town B is due east of town A, and that town C is due north of town B. Roger does some calculations and concludes that the distance from town A to town C is 430.116 miles. Should Roger leave his answer like that? Why or why not? If not, what answer should Roger give?

21. Suppose there are two rectangular pools: one is 30 feet wide, 40 feet long, and 3 feet deep throughout, the other is 20 feet wide, 40 feet long, and 5 feet deep throughout. Compare the sizes of the pools in *two* meaningful ways *other than* by comparing one-dimensional aspects.

22. Julie is confused about why we *multiply* by 3 to convert from yards to feet. She thinks we should *divide* by 3 because feet are *smaller* than yards. Explain in several different ways that why we multiply by 3 to convert from yards to feet.

23. Analyze the following calculations which intend to convert 25 square meters to square feet. Which ones use legitimate methods and are correct, and which are not? Explain.

(a)

$$25 \text{ m}^2 = 25 \text{ m } \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} = 82 \text{ ft}^2.$$

(b)

$$25 \text{ m}^2 = 25 \text{ m}^2 \times \frac{100 \times 100 \text{ cm}^2}{1 \text{ m}^2} \times \frac{1 \text{ in}^2}{2.54 \times 2.54 \text{ cm}^2} \times \frac{1 \text{ ft}^2}{12 \times 12 \text{ in}^2} = 269 \text{ ft}^2.$$

(c) 25 square meters is the area of a square that is 5 meters wide and 5 meters long.

$$5 \text{ m} = 5 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} = 16.404 \text{ ft}$$

Therefore,

$$25 \text{ m}^2 = 16.404 \times 16.404 \text{ ft}^2 = 269 \text{ ft}^2.$$

24. Johnny wants to calculate the perimeter of the shape shown at the top of Figure 9. Johnny's method is to shade the squares along the border of the shape, as shown in the bottom picture of Figure 9, and to count these border squares. Therefore Johnny says the perimeter of the shape is 24 cm. Is Johnny's method valid? If not, what is a correct way to calculate perimeter? How might you help Johnny better understand what perimeter is?

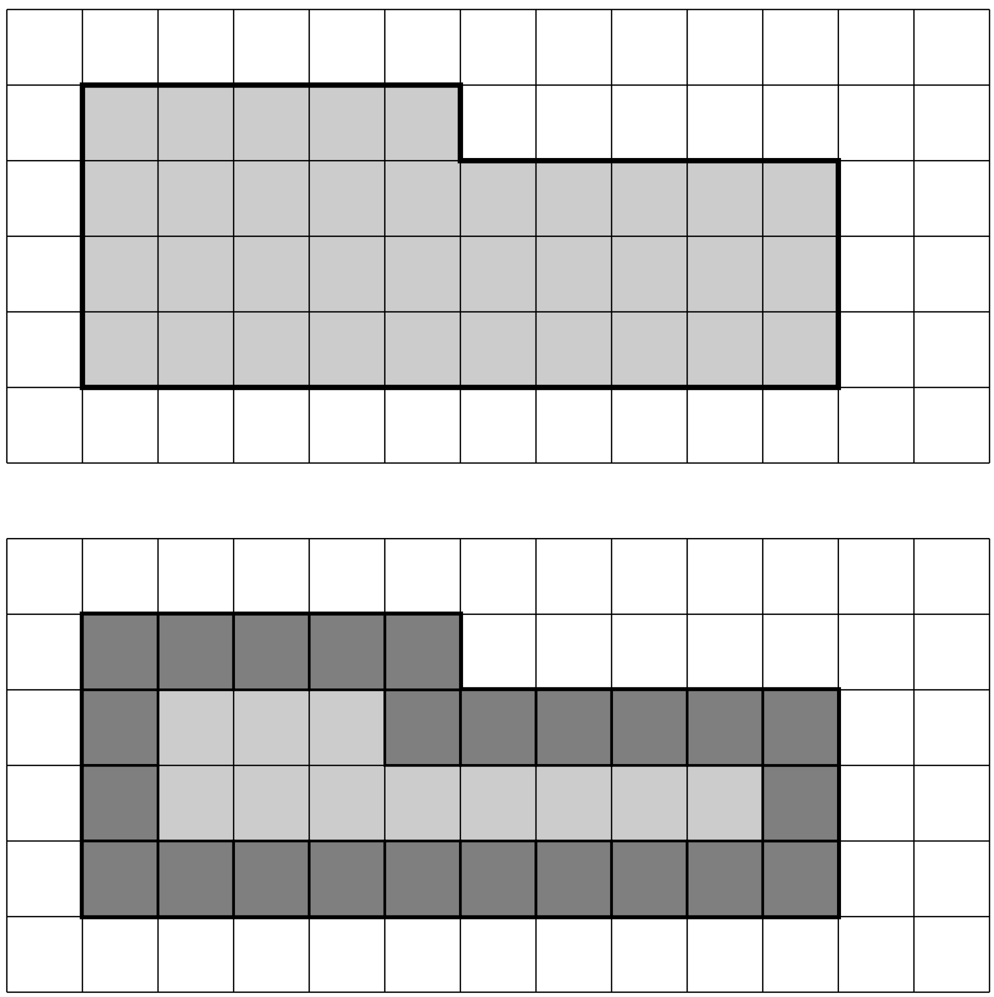


Figure 9: What is the Perimeter?

25. Determine the area of the triangle in Figure 10 using *no formulas other than the formula for the area of a rectangle.* Explain your reasoning. Adjacent dots on the grid are 1 cm apart.

26. What is the longest pole that can fit in a box that is 4 feet wide, 3 feet deep, and 5 feet tall? Explain your answer.

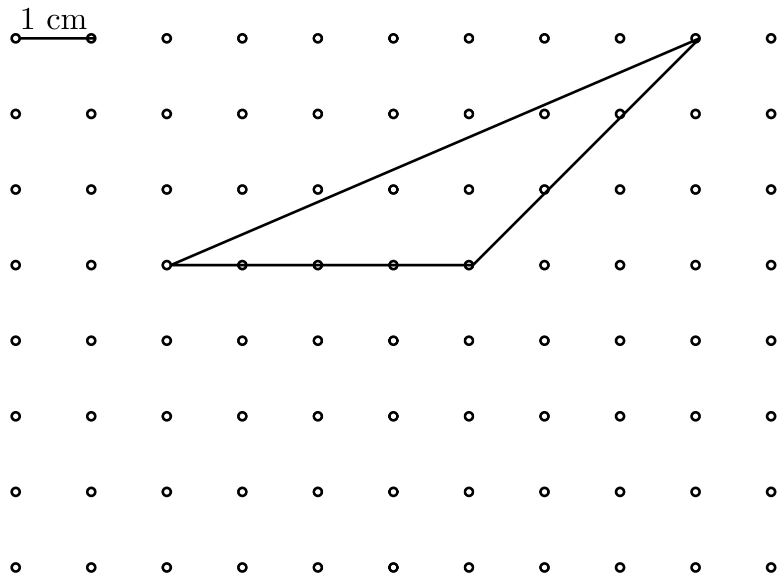


Figure 10: A Triangle

27. Explain clearly why there can be no formula for areas of parallelograms that is *only* in terms of the lengths of the sides of the parallelogram.

28. A sand and gravel company has a cone-shaped pile of sand. The company measures that the distance around the pile of sand at the base is 85 feet and the “slanted” distance from the edge of the pile at ground level to the top of the pile is 25 feet. Determine the volume of sand in the cone-shaped pile.

29. Nick wants to find the area of an irregular shape. Nick cuts a piece of string to the length of the perimeter of the shape. Nick measures that the string is about 40 cm long. Nick then forms his string into a square on top of centimeter graph paper. Using the graph paper, Nick determines that the area of his string square is about 100 cm^2. Nick says that therefore the area of the irregular shape is also 100 cm^2. Is Nick’s method for determining the area of the irregular shape valid or not? Explain. If the method is not valid, what can you determine about the area of the irregular shape from the information that Nick has? Explain.

30. A typical adult male gorilla is about $5\frac{1}{2}$ feet tall and weighs about 400 pounds. King Kong was supposed to have been about 20 feet tall. Assuming that King Kong was proportioned like a typical adult male gorilla, approximately how much should King Kong have weighed? Explain your reasoning.

Sources:

Instructor’s Manual for *Arithmetic for Elementary Teachers*, first edition, by Sybilla Beckmann, Addison-Wesley, 2004 (expected).

Mathematics for Elementary Teachers volume 2, preliminary edition by Sybilla Beckmann, Addison-Wesley, 2003.