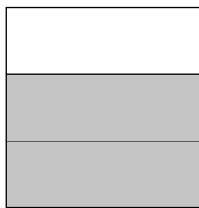


Sample Problems for MATH 5003, University of Georgia

1. Discuss the difference between dividing *in* half and dividing *by* $\frac{1}{2}$. Your discussion should include:
 - (a) a description of ways to express these two situations using mathematical notation,
 - (b) story problems for both situations; say which is which and explain how to solve the problems with the aid of pictures.
2. Write a *how many groups?* story problem for $2\frac{1}{2} \div \frac{3}{4}$ and explain how to solve the problem with the aid of pictures.
3. Tonya and Chrissy are working on $1 \div \frac{2}{3}$ by thinking about it as “how many $\frac{2}{3}$ cups of water are in 1 cup of water?” and by drawing diagrams. They draw a diagram of a large cup divided into three equal pieces, and they shade two of those pieces.



Tonya says: There is one $\frac{2}{3}$ cup of water in 1 cup and there is $\frac{1}{3}$ cup of water left over, so the answer should be $1\frac{1}{3}$.

Chrissy says: The part left over is $\frac{1}{3}$ cup of water, but the answer is supposed to be $1\frac{1}{2}$. Did we do something wrong?

Help Tonya and Chrissy.

4. Johnny is solving the multiplication problem 13.8×1.42 by multiplying longhand. Ignoring the decimal points, Johnny gets 19596, and now he must figure out where the decimal point goes. Explain how Johnny could use *reasoning about the sizes of the numbers* to figure out where the decimal point should go in his answer if he *doesn't know* the rule about adding the number of places behind the decimal points in 13.8 and 1.42.
5. As indicated in Figure 1, to get from 1.36 to 136 we multiply by 10×10 . To get from 2.7 to 27, we multiply by 10. In other words,

$$136 = 10 \times 10 \times 1.36 \quad \text{and} \quad 27 = 10 \times 2.7$$

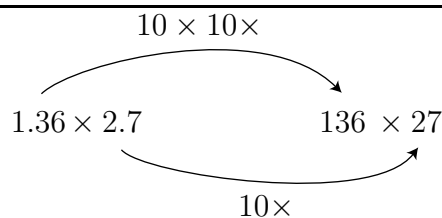
Therefore what should we do to

$$136 \times 27 = 3672$$

to get back to

$$1.36 \times 2.7?$$

Use your answer to explain the placement of the decimal point in 1.36×2.7 .



therefore:

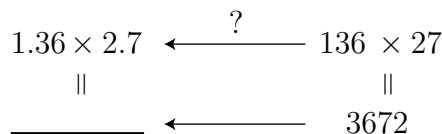


Figure 1: Comparing 1.36×2.7 to 136×27

6. To make a shade of orange paint that you like, you must mix $\frac{2}{3}$ of a bottle of red paint with each $\frac{4}{5}$ of a bottle of yellow paint that you use. You need 88 bottles of this orange paint. How many bottles of red paint will you need and how many bottles of yellow paint will you need? (All bottles are the same size.)
7. Which of the following two mixtures will make a more “lemony” drink?
 - A mixture of 2 parts lemonade crystals to 10 parts water, or
 - a mixture of 3 parts lemonade crystals to 11 parts water?

Jenny thinks that both mixtures will be equally “lemony” because you add one part of lemonade concentrate and also one part of water to the first mixture to get the second mixture. Jenny says these added parts “cancel” each other. Discuss Jenny’s reasoning, is it valid or not? If not, which drink will be more “lemony” and why?
8. A proportion problem: You need $1\frac{1}{2}$ cups of grape juice for a recipe that makes 4 servings. How many cups of grape juice will you need if you want to make 10 servings of the same recipe?
 - (a) Solve this proportion problem by setting up a proportion in which you set two fractions equal to each other.
 - (b) In terms of the story problem, what do the two fractions that you set equal to each other in part (a) represent, and why does it make sense to set these two fractions equal to each other?
 - (c) Why does it make sense to cross-multiply the two fractions in part (a)? What is the logic behind this procedure?
9. Suppose that you have two square garden plots: one is 10 feet by 10 feet and the other is 15 feet by 15 feet. You want to cover both gardens with a one inch layer of mulch. If the 10 by 10 garden took $3\frac{1}{2}$ bags of mulch, could you figure

out how many bags of mulch you'd need for the 15 by 15 garden by setting up a proportion:

$$\frac{3\frac{1}{2}}{10} = \frac{x}{15}?$$

Explain clearly why or why not. If the answer is no, solve the problem in another way if it is possible to do so. It may help you to draw pictures of the gardens.

10. (a) Explain what *multiples* and *factors* are, as if you were teaching these concepts to 4th or 5th graders in an introductory lesson.
(b) Describe an activity involving concrete objects that is designed to help children learn about *multiples*.
(c) Describe an activity involving concrete objects that is designed to help children learn about *factors*.
11. Johnny says that 3 is a multiple of 6 because you can put 3 cookies into 6 groups by putting $\frac{1}{2}$ of a cookie in each group. Discuss Johnny's idea: in what way does he have the right idea about what the term *multiple* means, and in what way does he need to modify his idea?
12. Explain why the Sieve of Eratosthenes produces a list of prime numbers.
13. The list of prime numbers up to 50 is as follows:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

- (a) Briefly describe how to use the above list of prime numbers to determine whether or not the number 2003 is prime. Determine if 2003 is prime or not.
- (b) Explain why you do not need any more prime numbers than are shown in the above list in order to determine whether or not 2003 is prime.
14. (a) Explain what the *least common multiple* is, as if you were teaching this concept to 4th or 5th graders in an introductory lesson.
(b) Describe an activity involving concrete objects that is designed to help children learn about least common multiples.
15. One definition of the term *even* is the following: a counting number is even if that number of objects can be divided into groups of 2 with none left over.
 - (a) What is another way that the term *even* could be defined for counting numbers *by referring to objects*? Why is this definition equivalent to the one above?
 - (b) What is a *numerical* way that the term *even* could be defined for counting numbers? Why is this definition equivalent to the one above?
 - (c) Is it possible to extend the definition of *even* to all integers? If so, is 0 even or not? Is -5 even or not?
 - (d) Is it possible to extend the definition of *even* to all rational numbers (fractions)? Explain why or why not.

16. Explain why the divisibility test for 3 is valid for 3-digit counting numbers in the following way. Let ABC represent a three digit whole number (A hundreds, B tens, and C ones). We can represent the number ABC physically with A bundles of 100 toothpicks, B bundles of 10 toothpicks, and C individual toothpicks. Think about dividing *each* of the A bundles of 100 toothpicks and *each* of the B bundles of 10 toothpicks into groups of 3. Use this to explain why it is valid to check if ABC is evenly divisible by 3 by checking if $A + B + C$ is evenly divisible by 3.

Relate your explanation to the following equations:

$$\begin{aligned} ABC &= A \cdot 100 + B \cdot 10 + C \\ &= (A \cdot 99 + B \cdot 9) + (A + B + C) \\ &= (A \cdot 33 + B \cdot 3) \cdot 3 + (A + B + C) \end{aligned}$$

17. Is it true that a whole number is evenly divisible by 6 exactly when the sum of its digits is divisible by 6? Investigate by trying a number of examples. State your conclusion.
18. Figure 2 shows a sequence of patterns made of small squares.

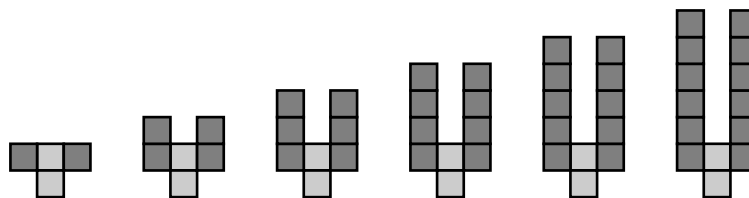


Figure 2: A Sequence of Patterns

- (a) Describe the sequence of patterns in words. How does the sequence grow?
- (b) Write a corresponding sequence of numbers by writing the total number of small squares in each pattern.
- (c) Imagine the sequence of patterns continuing forever, so that for each counting number N , there is an N th pattern. Write a formula for the number of small squares in the the N th pattern.
- (d) Relate your formula in part (c) to your description in part (a).
19. Assume that the pattern of shapes in Figure 3 continues to the right.

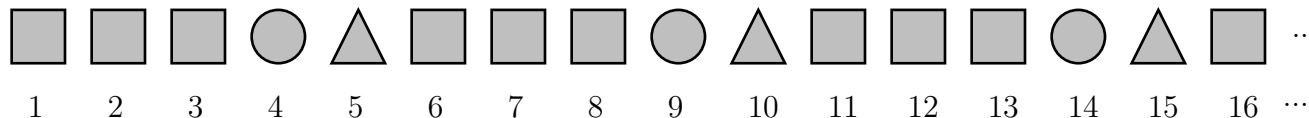


Figure 3: A Pattern of Shapes

- (a) What shape will be above the number 183? How can you tell?

- (b) Write two questions that you could ask children about the pattern in Figure 3 and that would help the children develop their thinking about patterns. Your questions should not be like question (a).
20. What is the ones digit of 2^{100} ? Explain how you can tell.
21. (a) Explain why the number of dots in pattern A of Figure 4 is equal to

$$1 + 6 \cdot 5 + 4 \cdot 3.$$

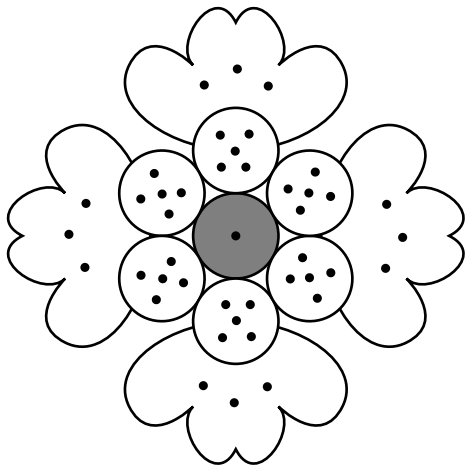
- (b) Find an expression involving multiplication and addition, like the expression in part (a), that is equal to the number of dots in pattern B of Figure 4. Explain why your expression describes the total number of dots.
- (c) Find expressions involving multiplication and addition that are equal to the number of dots in patterns C and D of Figure 4. Explain why your expressions describe the total number of dots in each case.
- (d) Now consider “general” flower patterns like the ones in Figure 4. If M , N , and P are any counting numbers, we can imagine a flower pattern that has M dots in the shaded circle, N dots in each of the white circles, and P dots in each of the “petals” of the flower. Write a formula, in terms of M , N , and P , for the number of dots in the flower pattern. Explain why your formula describes the total number of dots.
- (e) Create a design that illustrates the formula

$$M + 2N + 2P$$

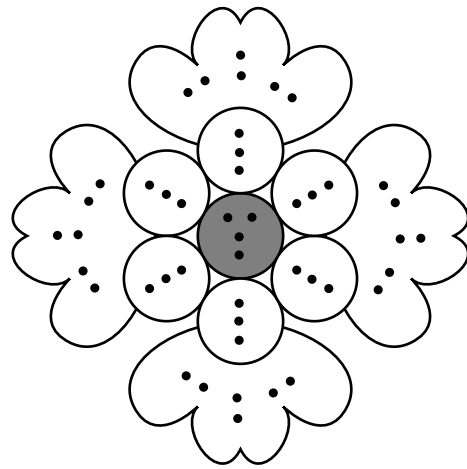
by imagining different portions of the design filled with different numbers of dots. Explain why your design illustrates the formula $M + 2N + 2P$.

22. The graph in Figure 5 shows Kelly’s distance from school while she is walking home. Write a story about Kelly’s walk home that fits with this graph. The features of the graph should be reflected in your story. Explain why your story fits with the graph.
23. Briefly discuss the use of line graphs: when is it appropriate to use a line graph and when is it not appropriate to use a line graph? Give two examples to illustrate.
24. The table below gives some information about children’s eating habits. Would it be appropriate to use a single pie graph to display this information? Explain your answer.

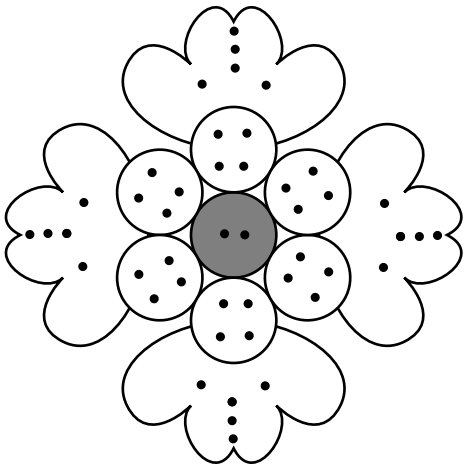
Food	Percent of 4–6 year olds meeting the dietary recommendation for the food
Grains	27%
Vegetables	16%
Fruits	29%
Saturated fat	28%



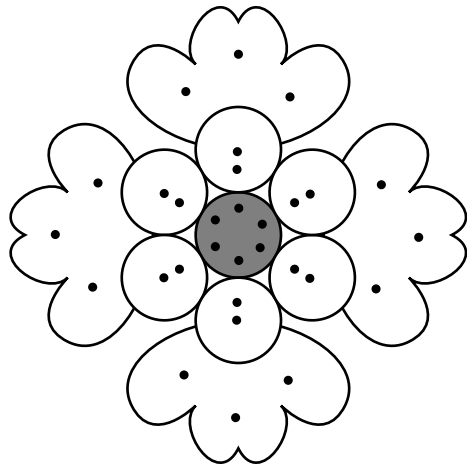
pattern A



pattern B



pattern C



pattern D

Figure 4: Flower Patterns

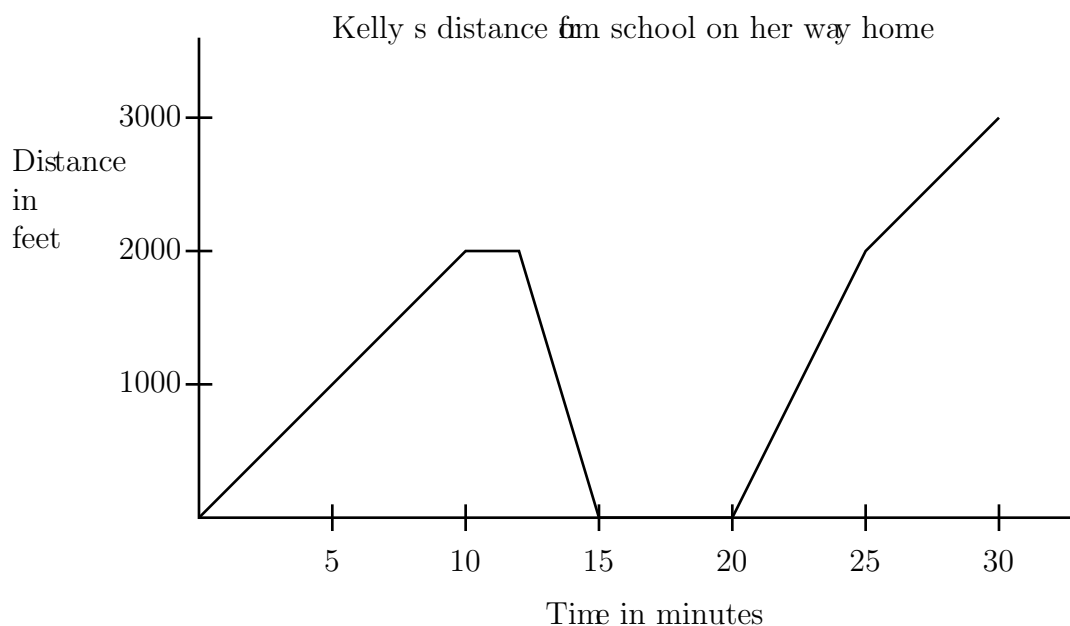


Figure 5: Kelly's Distance From School While Walking Home

25. Three 3rd grade classes are having a contest to see which class can read more pages during the month of February. The classes want to create a display, to be posted in the hall, that will show their progress.
 - (a) What display do you recommend? Be specific, and make sure your recommendation can realistically be carried out.
 - (b) Write at least three questions that you could ask the children about the display. Include at least one question for each of the three levels described by Frances Curcio (as described in a class activity): Reading the Data (RD), Reading between the Data (RBW), and Reading beyond the Data (RBY).
26.
 - (a) Explain what the mathematical term *average* or *mean* means.
 - (b) Describe a way that you could use concrete objects to help 4th or 5th graders learn about the mathematical concept of average.
27. Juanita read an average of 3 books a day for 4 days. How many books will Juanita need to read on the 5th day so that she will have read an average of 5 books a day over 5 days? Solve this problem in several ways and explain your solutions.
28. Suppose that all 4th graders in a state take a writing competency test that is scored on a 5 point scale. Is it possible for 80% of the 4th graders to score below average? If so, show how that could occur; if not, explain why not.
29. What is the difference between getting 90% correct on a test and being in the 90th percentile on a test? Give some specific examples to illustrate.
30. The questions below are about the following experiment: Suppose you put 10 pennies in a bag, shake the bag, and dump out the pennies.

- (a) Before you go on to the next questions, make a *guess*: what do you think is the probability of getting exactly 5 heads out of the 10 pennies when you dump the pennies out of the bag?
- (b) Let's say that everyone in a class of 25 does the experiment of dumping 10 pennies out of a bag 2 times, and that everyone writes each result on an index card. So now there are 50 index cards, each showing a number of heads and a number of tails that add to 10.
What is a good way to display these 50 pieces of data?
What would you expect the data display to look like, and why would you expect it to look like that?
- (c) Try the experiment of dumping 10 pennies out of a bag many times in a row, and display your data, using your suggestion in problem (2). Compare your results to your prediction in problem (2).
- (d) Is the probability of getting exactly 5 heads out of the 10 coins 50%? What does your data from problem (3) suggest?
31. Use the circle in Figure ?? (the circle is half blue, $1/4$ red, and $1/4$ yellow), a pencil, and a paperclip to make a spinner as follows. Put the pencil through the paperclip and put the point of the pencil on the center of the circle. The paperclip will now be able to spin freely around the circle.
- (a) What is the probability of the spinner landing on blue? Why?
- (b) To win a game, Jill needs to spin a blue followed by a red in her next two spins.
- i. Make a guess: what do you think Jill's probability of winning is?
 - ii. Carry out the experiment of spinning the spinner twice in a row 20 times. (In other words, spin the spinner 40 times, but each experiment consists of 2 spins.) Out of those 20 times, how often does Jill win? What fraction of 20 does this represent? Is this close to your guess in part (a)?
 - iii. Now calculate Jill's probability of winning theoretically as follows: Imagine that Jill carries out the experiment of spinning the spinner twice in a row many times. In the ideal, what fraction of those times should the first spin be blue?
In the ideal, what fraction of those times when the first spin is blue should the second spin be red?
Therefore, in the ideal, what fraction of pairs of spins should Jill spin first a blue, then a red? Therefore what is Jill's probability of winning?

Sources:

Mathematics for Elementary Teachers, preliminary edition by Sybilla Beckmann, Addison-Wesley, 2003.

Instructor's Manual for *Mathematics for Elementary Teachers*, first edition, by Sybilla Beckmann, Addison-Wesley, 2004 (expected).