

**MATH 5001/7001 Arithmetic and Problem Solving 3 hours**  
University of Georgia

Syllabus as of Spring 2005

Oasis Title: ARITH & PROB SOLV.

**Brief description:** A deep examination of topics in mathematics that are relevant for elementary school teaching. Problem solving. Number systems: whole numbers, integers, rational numbers (fractions) and real numbers (decimals) and the relationships between these systems. Understanding multiplication and division, including why standard computational algorithms work. Properties of arithmetic. Applications of elementary mathematics.

**Course Objectives:** To strengthen and deepen knowledge and understanding of arithmetic, how it is used to solve a wide variety of problems, and how it leads to algebra. In particular, to strengthen the understanding of and the ability to explain why various procedures from arithmetic work. To strengthen the ability to communicate clearly about mathematics, both orally and in writing. To promote the exploration and explanation of mathematical phenomena. To show that many problems can be solved in a variety of ways.

**Topical Outline:**

Problem solving: Polya's principles. Writing explanations.

Numbers: The natural numbers, the whole numbers, the rational numbers (fractions), and the real numbers (decimals). The decimal system and place value. Representing decimals with bundled objects. Representing decimals on a number line. Comparing sizes of decimals. Finding decimals in between decimals. Rounding decimals. The meaning of fractions. The importance of the whole associated with a fraction. Improper fractions. Equivalent fractions. Simplest form of a fraction. Fractions as numbers on number lines. Comparing sizes of fractions: by giving them common denominators, by converting to decimals, and by cross-multiplying. Using other reasoning to compare sizes of fractions. Solving fraction problems with the aid of pictures. Percent. Benchmark percentages and their common fraction equivalents. Solving percentage problems with the aid of pictures. Solving percentage problems numerically.

Addition and subtraction: Interpretations of addition and subtraction. The relationship between addition and subtraction. Explaining why the standard algorithms for adding and subtracting whole numbers and decimals work. Using regrouping in situations other than base 10, for example in calculating elapsed time by replacing 1 hour with 60 minutes. Adding and subtracting fractions. Explaining why we add and subtract fractions the way we do. The importance of the whole when adding and subtracting fractions, especially in story problems. Recognizing and writing story problems for fraction addition and subtraction. Recognizing story problems that are not solved by fraction addition or subtraction. Mixed numbers. Understanding when percentages should and should not be added. Calculating percent increase and decrease with the aid of pictures. Calculating percent increase and decrease numerically. Percent *of* versus percent increase or decrease. The commutative and associative properties of addition and their use in mental arithmetic. Using properties of addition to aid the learning of basic addition facts. Other (mental) methods for adding and subtracting: rounding and compensating, subtracting by adding on. Writing equations that correspond to a mental method of calculation (to demonstrate the connection between mental arithmetic and algebra).

Multiplication: The meaning of multiplication. Ways of showing multiplicative structure: with groups, with arrays, and with tree diagrams. Using the meaning of multiplication to explain why various problems can be solved by multiplying. Explaining why multiplication by 10 is easy in the decimal system. Why the commutative and associative properties of multiplication and the distributive properties make sense and how to illustrate them with arrays, areas of rectangles, and volumes of boxes. Using properties of arithmetic in solving arithmetic problems mentally. Writing equations that correspond to a mental method of calculation (to demonstrate the connection between mental arithmetic and algebra). Using properties of arithmetic to aid in the learning of basic multiplication facts. The distributive property and FOIL. Using multiplication to estimate how many. The partial products multiplication algorithm. Using pictures and the distributive property to explain why the standard and partial products procedures for multiplying whole numbers are valid. Explaining why non-standard strategies for multiplying can be correct or incorrect, such as explaining why  $23 \times 23 \neq 20 \times 20 + 3 \times 3$  and explaining why  $32 \times 28 = 30 \times 30 - 2 \times 2$ . The meaning of multiplication for fractions. Recognizing and writing story problems for fraction

multiplication. Recognizing story problems that are not problems for fraction multiplication. Explaining why the procedure for multiplying fractions works. Powers. (Optional: scientific notation.) Multiplication of decimals: explaining why the procedure for the placement of the decimal point is valid. Multiplication of negative numbers. Understanding that multiplication does not always “make bigger.”

Division: The meaning of division (two interpretations, with or without remainder). Understanding when the answer to a story problem solved by whole number division is best expressed as a decimal, as a mixed number, or as a whole number with a remainder. Why dividing by zero is undefined. The scaffold method of division. Explaining why the scaffold and standard longhand procedure for dividing whole numbers works. Explaining why some non-standard methods of division are valid. The relationship between fractions and division: explaining why  $\frac{A}{B} = A \div B$ . Calculating decimal representations of fractions. Explaining the relationship among remainder, mixed number, and decimal answers to division problems.

Text: Mathematics for Elementary Teachers, first edition, by Sybilla Beckmann, Addison-Wesley, 2005

Chapter 1 Problem Solving, Chapter 2 Numbers, Chapter 3 Fractions, Chapter 4 Addition and Subtraction, Chapter 5 Multiplication, Chapter 6 Multiplication of Fractions, Decimals, and Negative Numbers, Chapter 7 Division, through section 7.3 only.

**Math 7001:** for graduate credit, students must complete an additional course project. The project could consist of several essays, or a longer paper, in which the student discusses some aspect of the course material in depth, or in which the student relates the course material to their future teaching (e.g., with a collection of lesson plans or with a discussion of some lesson plans). However, other creative ideas could also be acceptable. For example, students might think of a creative way to tie their course project for math 7001 to something they will be doing for one of their other courses.