

MATH 5002/7002 Arithmetic and Problem Solving 3 hours
University of Georgia

Oasis Title: GEOM & PROB SOLV.

Prerequisite: MATH 5001

Brief description: A deep examination of topics in mathematics that are relevant for elementary school teaching. Geometric shapes and their properties. Measurement, especially length, area, and volume. Applications of elementary mathematics.

Course Objectives: To strengthen and deepen knowledge and understanding of measurement and basic geometry and how they are used to solve a wide variety of problems. In particular, to strengthen the understanding of and the ability to explain why various procedures and formulas in mathematics work. To strengthen the ability to communicate clearly about mathematics, both orally and in writing. To promote the exploration and explanation of mathematical phenomena. To show that many problems can be solved in a variety of ways.

Topical outline:

Visualization: Visualizing cross-sections of solid objects. Explaining why there are time zones. Explaining the phases of the moon. Using diagrams that show the Earth, Moon, and Sun rays as seen from outer space in order to determine the phase of the Moon and the time of day at a selected spot on the Earth.

Angles: angles as amount of rotation and as rays meeting at a point. How light reflects. Angles that the Sun's rays make at the Earth's surface. Proving Thales's theorem about the vertical angles that are created when two lines intersect.

Circles and spheres: definitions of circles and spheres. Ways that circles and spheres can intersect. Story problems about circles.

Triangles: definition of triangles. Showing that the sum of the angles in any triangle is 180° .

Quadrilaterals and other polygons: definitions of square, rectangle, rhombus, parallelogram, and trapezoid. Explaining how the various kinds of quadrilat-

erals are related. Understanding when relationships can be explained directly from the definitions and when information that is derived from the definitions, but not stated directly in the definitions, is needed. Showing relationships with Venn diagrams. Diagonals of quadrilaterals, especially rhombuses. Angles in quadrilaterals, especially rhombuses. The sum of the angles in an n -gon.

Constructions with straightedge and compass: bisecting a line segment and bisecting an angle. Relating the constructions to properties of rhombuses.

Polyhedra and other solid shapes: definitions of prisms, cylinders, pyramids, and cones. Making patterns for and analyzing prisms, cylinders, pyramids, and cones. The 5 Platonic solids. Explaining why there are no other Platonic solids.

Transformation geometry: reflections, translations, rotations, glide-reflections. Defining reflection symmetry, rotation symmetry, translation symmetry, and glide-reflection symmetry. Creating designs with specified symmetry. Optional: use of Geometer's Sketchpad.

Congruence: side-side-side triangle congruence and the structural stability of triangles. Optional: angle-side-angle congruence and the size of one's reflected face in a mirror.

Similarity: explaining how to solve similarity problems by using a scale factor or by considering "relative sizes." Angle-angle-angle criterion for triangle similarity. Determining distances and heights by using similar triangles.

Measurement: the concept of measurement. The U.S. Customary and Metric systems of measurement. Reporting and interpreting measurements (proper use of rounding). Measurable attributes of objects and explaining why different objects in a collection can be considered "biggest" depending on which attribute is used for comparison. Informally: dimension (1, 2, 3). The distinction between length, area, and volume. Explaining why we add to calculate perimeters and why we multiply to calculate areas of rectangles and volumes of boxes.

Converting measurements: by reasoning about multiplication and division and by "dimensional analysis." Common errors in converting measurements.

Principles underlying calculations of areas. Calculating areas using only the

principles and the area formula for rectangles. Using the principles to prove the Pythagorean Theorem. Using the pythagorean theorem.

Additional ways to determine areas: approximating areas. Cavalieri's principle about shearing and area.

Area formulas: explaining why the formula for areas of triangles is valid. Explaining why the area formula for parallelograms is valid. Understanding why there is no formula for areas of parallelograms in terms of the lengths of the sides of the parallelogram. Areas of circles and the number pi. Explaining why pi is between 3 and 4. Explaining why the area of a circle of radius r is πr^2 .

Area versus perimeter: understanding that perimeter does not determine area. The range of possible areas for a given perimeter.

Principles underlying calculations of volume. Determining volume by submersing. Optional: Archimedes's principle on floating objects. Understanding and using volume formulas for prisms, cylinders, pyramids, and cones.

The behavior of area and volume under scaling.

Text: Mathematics for Elementary Teachers, first edition, by Sybilla Beckmann, Addison-Wesley, 2005

Chapter 8 Geometry, Chapter 9 Geometry of Motion and Change, Chapter 10 Measurement, Chapter 11 More About Area and Volume

Math 7002: for graduate credit, students must complete an additional course project. The project could consist of several essays, or a longer paper, in which the student discusses some aspect of the course material in depth, or in which the student relates the course material to their future teaching (e.g., with a collection of lesson plans or with a discussion of some lesson plans). However, other creative ideas could also be acceptable. For example, students might think of a creative way to tie their course project for math 7002 to something they will be doing for one of their other courses.