

**MATH 5003/7003 Arithmetic and Problem Solving 3 hours**  
University of Georgia

Oasis Title: ALG & PROB SOLV.

Prerequisite: MATH 5002

**Brief description:** A deep examination of topics in mathematics that are relevant for elementary school teaching. Number theory, algebra and functions, including ratio and proportion, probability and statistics. Posing and modifying problems.

**Course Objectives:** To strengthen and deepen knowledge and understanding of probability and statistics, elementary number theory and algebra, and how they are used to solve a wide variety of problems. In particular, to strengthen the understanding of and the ability to explain why various procedures and formulas in mathematics work. To strengthen the ability to communicate clearly about mathematics, both orally and in writing. To promote the exploration and explanation of mathematical phenomena. To show that many problems can be solved in a variety of ways. To learn to pose and modify mathematical problems.

**Topical outline:**

Division of fractions and decimals: The meaning of division for fractions. Recognizing and writing story problems for fraction division. Understanding the distinction between dividing *by*  $\frac{1}{2}$  and dividing *in*  $\frac{1}{2}$ . Explaining why the “invert and multiply” procedure for dividing fractions is valid. Explaining why the procedure for placement of the decimal point in decimal division problems is valid. Understanding that division does not always “make smaller.”

Ratio and proportion: the meanings of ratio and proportion. Solving ratio problems using only multiplication, division, and logical thinking. Explaining the logic behind setting up proportions by setting two fractions equal to each other and solving these proportions by cross-multiplying. Understanding when problems can't be solved by a proportion. Optional: the Consumer Price Index.

Number Theory: definitions of factors and multiples and concrete problems that use and illustrate these concepts, definitions of greatest common factor

and least common multiple and concrete problems that use and illustrate these concepts. Prime numbers. The Sieve of Eratosthenes for producing lists of prime numbers. The trial division method for determining if a number is prime. Factoring counting numbers into products of prime numbers. Optional: the proof that there are infinitely many prime numbers. Lightly: consequences of the irrationality of the square root of 3 for making designs with standard school pattern tile sets. Even and odd: different ways of defining *even* and their equivalence. Divisibility tests: explaining the divisibility tests for 2, 3, 5, and 9.

Algebra and functions: patterns, sequences, formulas, and equations. Writing expressions and equations to go along with a scenario. Evaluating expressions. Solving algebra word problems with Singapore strip diagrams. Using the imagery of a pan balance to understand the technique for solving linear equations in one variable. Arithmetic and geometric sequences. Creating numerical sequences from picture sequences, creating picture sequences from numerical sequences. Describing sequences in words and with formulas. Determining a specified entry (say, the 100th) in a repeating pattern. Optional: arithmetic and geometric series. Functions and their graphs. Relating qualitative descriptions of functions to their graphs. Understanding that the graph of a function is a line exactly when there is a fixed increase in the output for a given fixed increase in the input.

Basic descriptive statistics: Lightly: designing investigations and gathering data. Common ways to display data. Three levels of questions about graphs: reading the data, reading between the data, reading beyond the data. The average of a numerical set of data. Understanding the average as “making even” or “leveling out.” The median of a numerical set of data. Showing that different data sets can have the same median but a different average. Showing that different data sets can have the same average but a different median. Understanding that “more than half can be above average.” Percentiles. Understanding the difference between percentile and percent correct.

Probability: Basic principles of probability. Simple probability calculations. Using the meaning of fraction multiplication to understand simple probability calculations.

Critique of mathematics lessons: students should critique several mathematics lessons that they taught to elementary school children and discuss ways

that the lessons could be improved or extended. The critique should focus on mathematics content.

Solving, posing and modifying problems: Because fractions, decimals, and percents are traditionally difficult topics to teach, it is recommended that part of the course be devoted to solving, posing, and modifying problems involving fractions, decimals, and percents, especially problems that are relevant to and helpful for the teaching of these topics.

Text: Mathematics for Elementary Teachers, first edition, by Sybilla Beckmann, Addison-Wesley, 2005

Chapter 7 Division, sections 7.4 - 7.6 only, Chapter 12 Number Theory, Chapter 13 Functions and Algebra, Chapter 14 Statistics, Chapter 15 Probability.

**Math 7003:** for graduate credit, students must complete an additional course project. The project could consist of several essays, or a longer paper, in which the student discusses some aspect of the course material in depth, or in which the student relates the course material to their future teaching (e.g., with a collection of lesson plans or with a discussion of some lesson plans). However, other creative ideas could also be acceptable. For example, students might think of a creative way to tie their course project for math 7003 to something they will be doing for one of their other courses.