Enacting Mathematical Practices in a Community of All Mathematics Teachers

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Enacting Mathematical Practices

How do we enact a vision of mathematics teaching in which mathematical processes, proficiencies, and habits of mind play a central role?

Enacting Mathematical Practices

The "Standards Movement" is rooted in a vision of deeper engagement in mathematical thinking:

- Mathematical Process Standards (NCTM)
- Mathematical Proficiency (NRC)
- Mathematical Practices (CCSSM)

We can view the Standards Movement as a yearning for a community to take charge of mathematics teaching and to enact these practices.

Enacting Mathematical Practices requires community

Habits of mind can develop in a community in which the habits are the norm.

I will argue that:
We need a community of mathematics teachers, from Prekindergarten through the college level, who discuss their work in depth, build on each other's ideas, and work towards a shared view of excellent mathematics teaching.

But why don't we already have this community?
And how do we develop this community?

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How do we develop the community that enacts mathematical practices?

I will argue that:
Opportunities to gain peer recognition and high standing within the community *can* drive excellence.

External measures of assessment or oversight *don't* drive excellence.

I will explain how research in motivation informs us on these points.

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Why a community of *all* math teachers?

The time is right for *all of us who teach math* to join together to become community:

- Math teaching is connected across all levels;
- we have the Common Core in common;
  the Common Core gives us a chance to "shake things up;"
- *all of us* have much to learn from each other about both math and math teaching;
- vibrant communities exist, let's join them together for greater power.

We have a common problem in math teaching

- The current system is not fostering excellence in math teaching *at any level*;
- math teaching is not regarded as a top-notch profession *at any level*;
- math teaching is not vigorous and vibrant the way some other mathematical professions are, such as math research.
- information that could lead to improvements doesn't seem to "take hold"
  - knowledge about mathematics,
  - knowledge about teaching and learning.

Why is it like this?

Practices must grow from *within* a community

Example of how “external ideas” do not take root:

- Well-established research from decades ago that should inform math teaching is not as widely used as one would hope.

Examples of how “internal ideas” spur productive activity:

- In math research, important new theorems are immediately studied carefully and used in new results.
- In Japan, Lesson Study has been instrumental in producing a common vision of good teaching.

Practices must grow from *within* a community

"Health systems worldwide are increasingly aware of the need to modernise service delivery and medical practice. While innovation is one road to modernisation, there also already exist mountains of clinical evidence and many examples of managerial improvements that have not yet become standard practice."

"Viewing the process as one of 'spread' implies a pushing out of ideas; a doing unto, by someone who knows more, to someone else. This model engenders resistance that slows change. This is largely due to the difference in view between those pushing out the change, the spreaders, and those receiving the demands, the potential adopters." (Fraser and Plsek, 2003)

What can we learn from each other?

Example from fractions:

Mathematics educators: fractions appear in different guises, different kinds of situations, such as part-whole, division, operator, measurement, ratio.

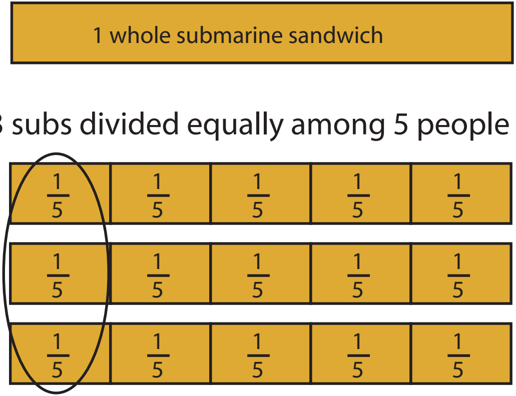
Mathematicians: Mathematics requires definitions. Starting from one definition, we can tie the other views together by lines of reasoning.

How are

$$\frac{A}{B}, \quad A \div B, \quad A : B$$

different but related?

Linking division and fractions



$$3 \div 5 = \frac{3}{5}$$

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$

1 person's share is $3 \div 5$

1 person's share is 3/5 of a sub

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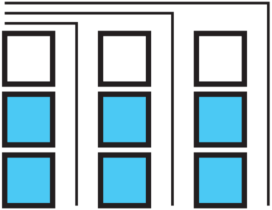
Fraction and ratio are distinct

Equivalent ratios



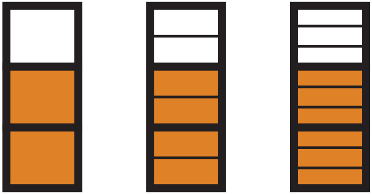
cups blue	2	4	6
total cups	3	6	9

more parts, same size parts

→

more total paint
more blue pigment

Equivalent fractions



$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

more parts, smaller parts

→

same whole amount
same portion

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Unit rates connect ratios to fractions

5 cups grape juice for every 2 cups peach juice.

cups grape	cups peach
5	2
5/2	1
1	2/5

Unit rates:
5/2 cups grape juice for every **1** cup peach juice;
2/5 cups peach juice for every **1** cup grape juice.

What can we learn from each other?

Mathematics educators:

- Good teaching requires trying to figure out how your students are thinking;
- learning *requires* the learner to make sense of the ideas for herself or himself;
- interactions with peers can help learners work through ideas.

We need to apply the above to ourselves as math teachers:

- We all need to make sense for ourselves of existing knowledge about how we might improve math teaching.
- We all need to be part of creating and sharing knowledge about how to teach math better.

What can we learn from each other?

PreK, Elementary, Middle School, High School, College teachers: share what our students are doing and how our students are thinking.

Research on motivation

Internal motivation is fostered by environments that fulfill the basic human needs for

- autonomy
- competence
- relatedness

as opposed to environments that are externally controlling.

Research on motivation

“The picture that emerged from these meta-analyses of 128 well-controlled experiments exploring the effects of extrinsic rewards on intrinsic motivation is clear and consistent. In general, tangible rewards had a significant negative effect on intrinsic motivation for interesting tasks, and this effect showed up with participants ranging from preschool to college, with interesting activities ranging from word games to construction puzzles, and with various rewards ranging from dollar bills to marshmallows.”
(Deci, Koestner, and Ryan, 1999)

Research on motivation

"…contingent, tangible rewards and other extrinsic factors such as competition and evaluations can be detrimental to outcomes such as creativity, cognitive flexibility, and problem solving which have been found to be associated with intrinsic motivation …"
(Gagne and Deci, 2005)

Research on motivation

"...work climates that promote satisfaction of [autonomy, competence, relatedness] will ...yield the important work outcomes of (1) persistence and maintained behavior change; (2) effective performance, particularly on tasks requiring creativity, cognitive flexibility, and conceptual understanding; (3) job satisfaction; (4) positive work-related attitudes; (5) organizational citizenship behaviors; and (6) psychological adjustment and well-being."
(Gagne and Deci, 2005)

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Research on motivation

"...pressures from schools, communities, and society for teachers to be more accountable for students' achievement can lead teachers to be more controlling and thus can be counterproductive for the goals of conceptual understanding and personal growth."
(Deci, Vallerand, Pelletier, and Ryan, 1991)

How can math teaching become a vigorous, vibrant profession that enacts mathematical practices?

What makes math research so strong and productive?

- **Public, shared knowledge**
  Math researchers publish their work, they discuss their work in depth, they build on each other's work;
  all can participate in publishing;
- **A reward structure of *peer approval***
  Quality is judged from *within* the community by peer recognition and admiration;
  the community is a meritocracy that is led by and learns from the top professionals in the field.
- **A high bar to entry**
  Entry into the community requires a high level of education and accomplishment.

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How could a community of all math teachers work together towards enacting mathematical practices?

All of us who teach math:

- Let's develop repositories of shared, vetted knowledge about math and its teaching;
- let's compete for each other's admiration through the sharing of ideas;
- let's develop a system that allows knowledge about math and its teaching to build and evolve;

What kind of shared, vetted knowledge might we produce together?

- Short essays (like a blog post) that are revised in response to comments from the community;
- Video clips (or transcripts) of teaching and accompanying discussion/commentary;
- Commentary on existing resources;
- Sample tasks for students;
- Other ideas?

Importantly:
These must be vetted by the community and they must be publicly available.
We must develop a culture *among all of us who teach math* of sharing knowledge about math teaching and building on the knowledge of others.

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Example from my class: writing equations

Some students (prospective teachers) state the commutative property of multiplication this way:

If $A \times B = C$*, then* $B \times A = C$*.*

This is correct, but why don't students just write it this way?

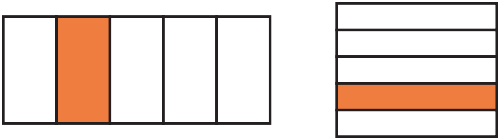
$A \times B = B \times A$ *for all numbers* A, B*.*

Example from my class: writing equations

Does it indicate that these students may be viewing $=$ as “becomes” or “evaluates to” rather than “is the same amount as”?

What makes this problem so difficult?

The diagram below is a map of Peter's garden. The two plots in Peter's garden have each been divided into 5 pieces of equal area. The shaded parts show where carrots have been planted.



1. What fraction of the area of Peter's garden is planted with carrots?
2. Use the CC definition of fraction to explain why your answer in part (1) is correct.
3. Explain in another way why your answer in part (1) is correct.

CC definition of fraction

Understand a fraction $\frac{1}{B}$ as the quantity formed by 1 part when a whole is partitioned into B equal parts;
understand a fraction $\frac{A}{B}$ as the quantity formed by A parts of size $\frac{1}{B}$.

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Which mathematical ideas and practices will be helpful for this question?

Why is the sun hotter when it's overhead rather than when it's lower in the sky?

Which mathematical ideas and practices will be helpful for this question?

There is fixed energy, E, in "a bundle" of sun rays;
the rays hit a smaller or larger area, A.

$$\frac{E}{A} \quad \frac{Joules}{m^2}$$

Compare to:
Which is greater $\frac{7}{39}$ or $\frac{7}{38}$?

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Example: thinking qualitatively about quantities

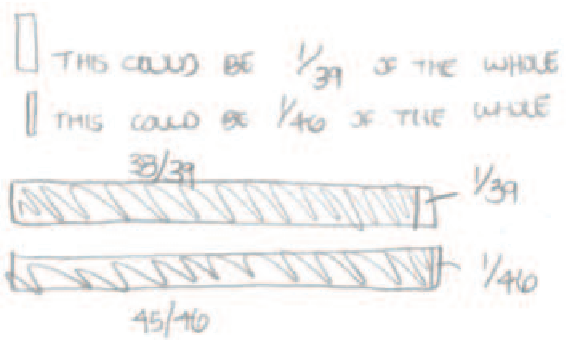
I find this kind of problem “high leverage” for prospective elementary and middle grades teachers because it encourages “qualitative thinking about quantities”:

Determine which is greater $\frac{38}{39}$ or $\frac{45}{46}$.

Which mathematical practices are involved in solving this?

Example: thinking qualitatively about quantities

MP 2: "…Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; …attending to the meaning of quantities, not just how to compute them; …"



Example: thinking qualitatively about quantities

If you compare $\frac{38}{39}$ to $\frac{45}{46}$ you can determine that $\frac{45}{46}$ is larger. The reasoning behind this is you n see that both fractions are one part away from being a complete whole, so then you have to look at the sizes of the parts. Fractions describe equal parts of a whole so the greater the denominator, the more parts the whole is split into. Every time you split a whole into more parts, those parts become smaller. This can easily be seen with the fractions $\frac{1}{2}$ and $\frac{1}{10}$. Even though each fraction is only one part of the whole their sizes vary greatly. For exampl

Graph of $\frac{1}{2}$

Graph of $\frac{1}{10}$

... problem, a whole split

Some projects for shared, vetted knowledge

- Illustrative Mathematics Project for the CC (Bill McCallum);
- Q&A site: Mathematics Teacher Educators, hosted by “Stack Exchange” (google “Mathematics Teacher Educators Area 51 Stack Exchange” or email me);
- Are you interested in helping to pilot a site to submit and host shared, vetted knowledge about mathematics teaching? email me at sybilla at math.uga.edu

Thank you to Jim!

A big thank you to Jim Lewis:

- for highlighting the importance of mathematical habits of mind;
- for creating a vibrant mathematics teaching community;
- for his leadership in showing how mathematicians can be engaged in mathematics education and work with mathematics educators;
- for his support of those of us who are interested in mathematics education!

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