

# Stable homotopy and Seiberg-Witten theory

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1. Review of SW-theory
2. The stable homotopy picture
3. Results - old and new
4. A more global view

## Review of Seiberg-Witten theory

$X$  4-manifold, closed,  $K$ -oriented

( $K$ -oriented  $\Leftrightarrow$  oriented +  $\text{Spin}^c$ -structure

$\Leftrightarrow$  oriented +  $S^+, S^-$   $\text{rk}_\mathbb{C}$ -2-bundles,

with  $\text{Hom}_\mathbb{C}(S^+, S^-) \cong T_\mathbb{C}^* X$ )

$M$  Moduli space of monopoles:

solutions to some PDE mod equivalence,

depends on: Riemannian metric  $g$ ,

selfdual 2-form  $\lambda \in \Omega_+^2(X)$

Properties of  $M$ :  $\exists b_+ > 0$ ,  $(g, \lambda)$  generic

$\Rightarrow M$  compact, orientable mfd, equipped with  
 $L$  complex line bundle.

$$SW(X) = c_1(L)^{\frac{\dim M}{2}} \cap [M] \in \mathbb{Z}$$

"characteristic number"      specify  $\uparrow$  orientation

