Rob Kirby

1961 14 Aug – 8 Sept "Topology of 3-Manifolds"
(Little red book)
Edited M K Fort Jr.

Decomposition spaces
n-Manifolds — M. Brown ∂M is collared
Stallings $F \to M^3 \to S^1$
Knots Fox - Quick trip.
Poincaré Conjecture Zeeman + Stallings for $n \geq 5$
Periodic Maps.

1967 1 week
1968 1 week

1969 11–22 August "Topology of Manifolds"
Ed. Cantrell + C H Edwards

Triangulations – 2 by Siebenmann
3-Dimensions + knots
Group actions
Embedding + Unknotting in Higher Dimensions
Surgery

Participants in 61

R.D. Anderson
JJ Andrews
BJ Ball
RH Bing
TR Brahana
* M Brown
S S Cairns
Ed Connell
ML Curtis
P Doyle
C.H. Edward Jr
* D.B.A. Epstein
MK Fort Jr.
R H Fox
* Herman Gluck
O.G. Harrold Jr.
T Homma
J. Kister
S. Kinoshita

G.R. Livesay
L.F. McAuley Jr.
E.E. Moise
R.H. Rosen
D.E. Sanderson
John Stallings
G.T. Whyburn
Lucille Whyburn
R.L Wilder
E.C. Zeeman

* Hempel
Cantrell

* Not retired

1977 1-12 August "Geometric Topology"
Ed Cantrell

Low dimensional manifolds
Topology of manifolds - Sullivan - Lipschitz
Shape theory & ∞-dim topology - Chapman

1985 5-16 Aug "Geometry & Topology"
Ed McCrory & Shifrin

Some gauge theory, but not too much low-dim topology due to MSRI year in 84-85.

1993 2-13 Aug "Geometric Topology"
Ed W. Kazez

Mostly low dim top.

$$\tau_4(M) = \sum_{\theta} \rho^{\mu(M,\theta)}$$ Paul Melvin

where $\rho = e^{2\pi i(-3/16)}$,

θ = spin structure on 3-manifold M "closed, oriented"

μ = μu-invariant $\in \mathbb{Z}/16$

Let (M,θ) be spin boundary of W^4.

Then $\mu(M,\theta) = \sigma(W^4)$ = signature $W^4/16$

This is independent of choice of W because

$$\sigma(X^4) \equiv 0 \ (16)$$

if X^4 is smooth, closed, spin. (Rohlin)

A spin structure θ is a trivialization of T_M over the 1-skeleton which extends over the 2-skeleton.

$$\{\theta\} \xleftrightarrow{1-1} H^1(M;\mathbb{Z}/2) \cong H_1(M;\mathbb{Z}/2)$$

$\exists$? polynomial time algorithm to evaluate τ_4 ? (M Freedman)

Probably not !

The number of spin structures on M,
2^{β_1}, grows exponentially in $\beta_1 = \text{rank}\, H_1$
$= |H_1(M;\mathbb{Z}/2)|$

Don't want to find different spin W^4 for each θ on M^3,
so fix W^4 + then

$$\mu(M,\theta) = \sigma(W) - F\cdot F + 8\,\text{Arf}(F) \quad (16)$$

where F^2 is a smooth embedded, <u>orient</u> surface in W^4 such that θ extends across $W - F$ but not across any component of F.

F is P.dual to w_2

Arf(F) is the Arf invariant of a quadratic enhancement on $H_1(F;\mathbb{Z}/2)$

$$\left(\begin{array}{l}\text{If } F \text{ non-orientable, then} \\ \mu(M,\theta) = \sigma(W) - F\cdot F + 2\,\mathrm{Brown}(F)\ (16)\end{array}\right)$$

How to find F? How to see θ?

Let W^4 be determined by a framed link L, n-components

Spin structures θ on $M = \partial W$ correspond to "characteristic" sublinks C which satisfy $C \cdot L_i \equiv L_i \cdot L_i \ (2)\ \forall L_i \in L$

"linking" "framing"

Construct F from cores of 2-handles attached to components of C, together with an oriented Seifert surface for C.

Now $\tau_4(M) = \sum_{\Theta = C} \rho^{\sigma(W) - F\cdot F + 8\,\mathrm{Arf}(F)}$

$\sigma(W)$ is a one-time calculation

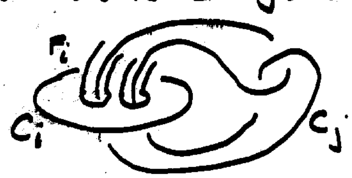
$F \cdot F$ is "quadratic", does not depend on F but only on linking form, & is <u>not</u> a problem.

But $\mathrm{Arf}(F) = \mathrm{Arf}(C)$ does depend on topology of F & we don't want to construct an F for each Θ = char. link C.

Need "local" definition of $\mathrm{Arf}(F)$.

Given C, suppose $lk(C_i, C_j) = 0$ $\forall C_i, C_j \subset C$

Then each C_i bounds a surface F_i which is disjoint from the other C_j'



Then $\mathrm{Arf}(c) \overset{2}{\equiv} \sum_i \mathrm{Arf}(c_i) - \sum_{ij} h(c_i, c_j) + \sum_{ijk} T(c_i c_j c_k)$

$$\mathrm{Brown}(M_c) \overset{8}{\equiv} \sum_i \mathrm{Brown}(c_i) - 2\sum_{ij} h(c_i, c_j) + 4\sum_{ijk} T(c_i c_j c_k)\Big)$$

where $T(c_i, c_j\, c_k)$ = number, mod 2, of
triple points in $F_i \cap F_j \cap F_k$.
$h(c_i, c_j)$ = framing, mod 2, of $F_i \cap F_j$
given by normals (to $F_i \cap F_j$) into F_i or F_j

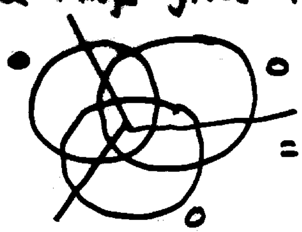
So, computing τ_4 in polyn time
comes down to evaluating

$$\sum_c \rho^{8\sum T(c_i c_j c_k)} = \sum_c (-1)^{\sum T(c_i c_j c_k)}$$

since $\mathrm{Arf}\, C_i$ is a one time calculation
for each component of L, + h is
"quadratic".

0-surgery on Borromean rings gives T^3

T^3 has 8 spin structures



$= L$

For 7, $\mu = 0$

For spin structure $C = L$,

$\mu = 8 = 8 \operatorname{Arf}(C) = 8\,T(L_1, L_2, L_3)$

so $\tau_4 = 7 - 1 = 6$.

Let $g = \sum_{i \neq j \neq k} a_{ijk}\, x_i \cdot x_j \cdot x_k$ be a cubic form over $\mathbb{Z}/2$ with n variables $a_{ijk} \in \mathbb{Z}/2$

Let L_g be the n strand unbraid but with the Borromean rings $(\sigma_1 \sigma_2^{-1})^3$, "tied" in the braid in the $i, j, + k$ strands for each $a_{ijk} = 1$.

Then $\tau_4(M_g) = \sum_C \rho^{8A} = \sum_C (-1)^A$ for $A = \sum_{c_i c_j c_k \in C} a_{ijk}$

$\sum_{(\mathbb{Z}/2)^n} (-1)^{g(x_1 \cdots x_n)}$ $\quad (x_1 \cdots x_n) \in (\mathbb{Z}/2)^n$

Evaluating $\sum_{(x_i)\in F} (-1)^{g(x_1 \cdots x_m)}$ is #P-complete

P A problem is in the class P if $\exists$ a polyn time algorithm giving a solution

NP A problem is NP (non-deterministic polyn time) if, e.g. a conjectured solution can be checked in polyn. time.

#P An NP problem is in #P if $\exists$ polyn time algorithm to determine the number of solutions.

3-Satisfiability is a universal #P problem
3 SAT

To count solutions to expressions
$E = A_1 \wedge A_2 \wedge \cdots \wedge A_m$ where each A_k is of the form $(x_{i_1}^{\pm 1} \vee x_{i_2}^{\pm 1} \vee x_{i_3}^{\pm 1})$.

Each x_i is assigned T or F & want E to be True.

$$3\text{-SAT} \equiv \sum (-1)^{g(x_1 \cdots x_m)}$$

Change E to system of equations over $\mathbb{Z}/2$ by letting $T=1$, $F=0$, e.g.

$$(x_1 \vee x_2 \vee \bar{x}_3) \longrightarrow (1-x_1)(1-x_2)x_3 = 0$$

Change these cubic equations (one for each A_i) to quadratic equations by letting a new variable $x_{ij} = x_i \cdot x_j$

Thus $(1-x_1)(1-x_2)x_3 = 0$ becomes

$$x_{12} - x_1 \cdot x_2 = 0 \quad \& \quad (1-x_1-x_2+x_{12})x_3 = 0$$

So 3-SAT is equivalent to counting the number of solutions over $\mathbb{Z}/2$, $\#X$, to k quadratic equations in m variables

$$X = \begin{cases} g_1(x_1 \cdots x_m) = 0 \mod 2 \\ \quad \vdots \\ g_k(x_1 \cdots x_m) = 0 \mod 2 \end{cases}$$