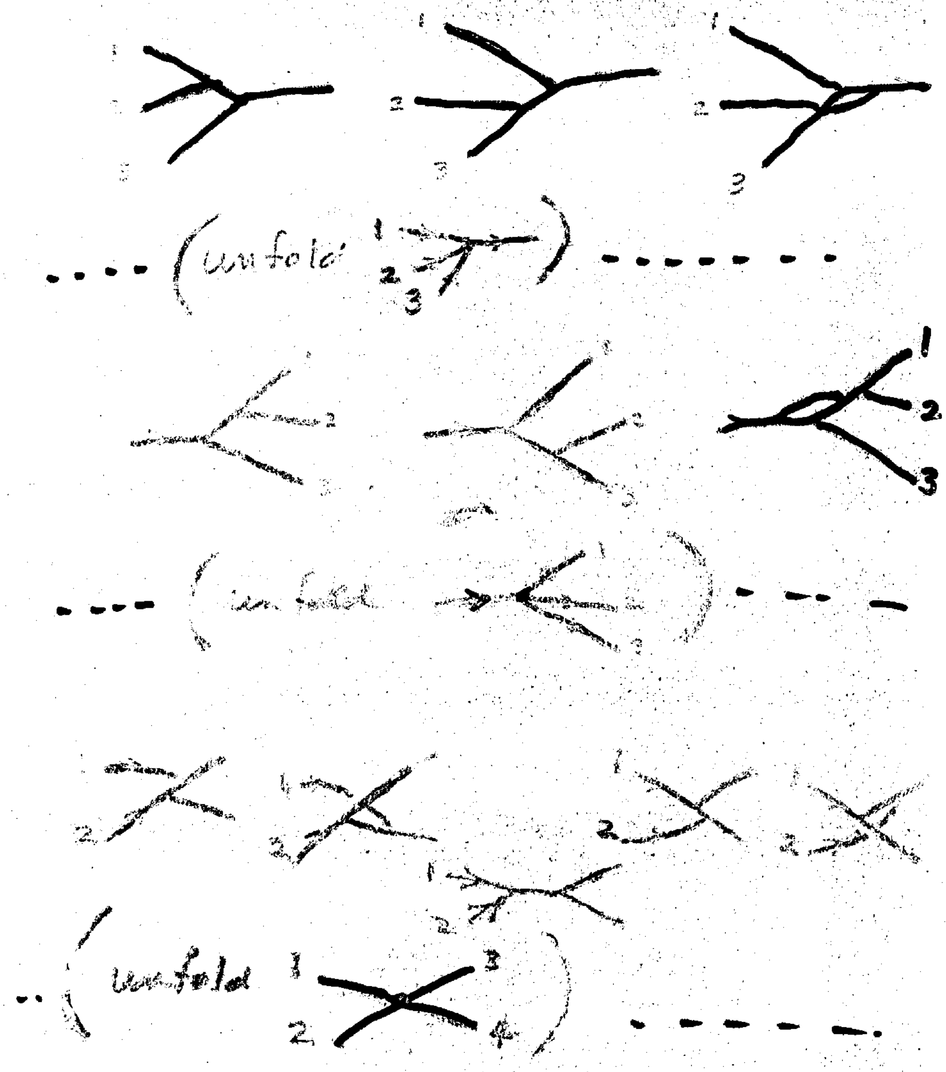
Dennis Sullivan

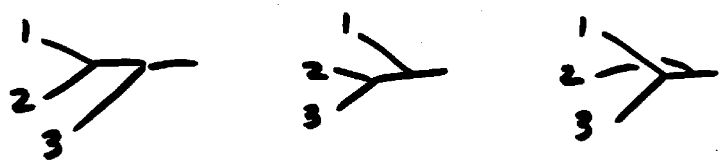


Suppose we have two operations on a space indicated by

 and 

The compositions  indicated are the relations Jacobi, coJacobi, compatibility of a Lie bialgebra

Example



joint with Moira Chas GT 9911159

(reduced)
<u>The</u> string homology of a manifold is a Lie bialgebra

For manifolds of dim 2 this is intersection theory of curves

Goldman Turaev 85, 98 ...

Algorithms

M. Chas GT 0105178

programs

Counterexample to a conjecture of Turaev

Combinatorial proof of Goldman theorem and generalization

~~Lets do~~ try to
Lets do the same for mappings of surfaces into the manifold :

$\longrightarrow$ $i+j+4-d+1$ cycle

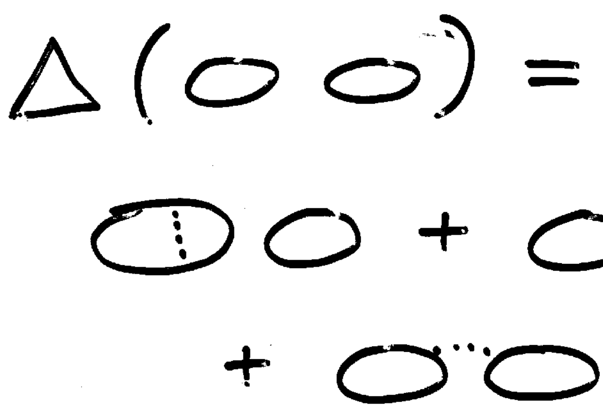
Jacobi works

Thm (comm) membrane homology of a manifold is a Lie algebra.

Note genus zero is a sub Lie algebra

self intersections are different for membranes and strings. instead of disconnecting the genus increases. unary Δ operator

extend operations to dis-connected surfaces



$\Delta \cdot \Delta = 0$. $[\,,\,]$ Leibniz

Thm. (total) membrane homology of a manifold ~~has~~ wants to be a product $\bullet$ and a differential Δ satisfying the axioms of a BV algebra. //

($\bullet$, Δ generate a $\bullet$ compatible bracket)

Note: a variant is true at the chain level using $\Delta' = \partial + \Delta$

Batalin and Vilkovisky are physicists who formulated an algebraic discussion, associated to any classical Lagrangian and going toward perturbative quantum field theory.

In this discussion a classical action is deformed to a new solution of

$$\Delta X + [X, X] = 0$$

"quantum action equation" ??

Are there natural solutions of this equation in the BV algebra of membranes in our manifold

Yes, if M admits a symplectic structure. Choose ω, J and consider (the) (a) Gromov chain associated to (limited) J-holomorphic curves in M.

(limit necks, concentration of energy)

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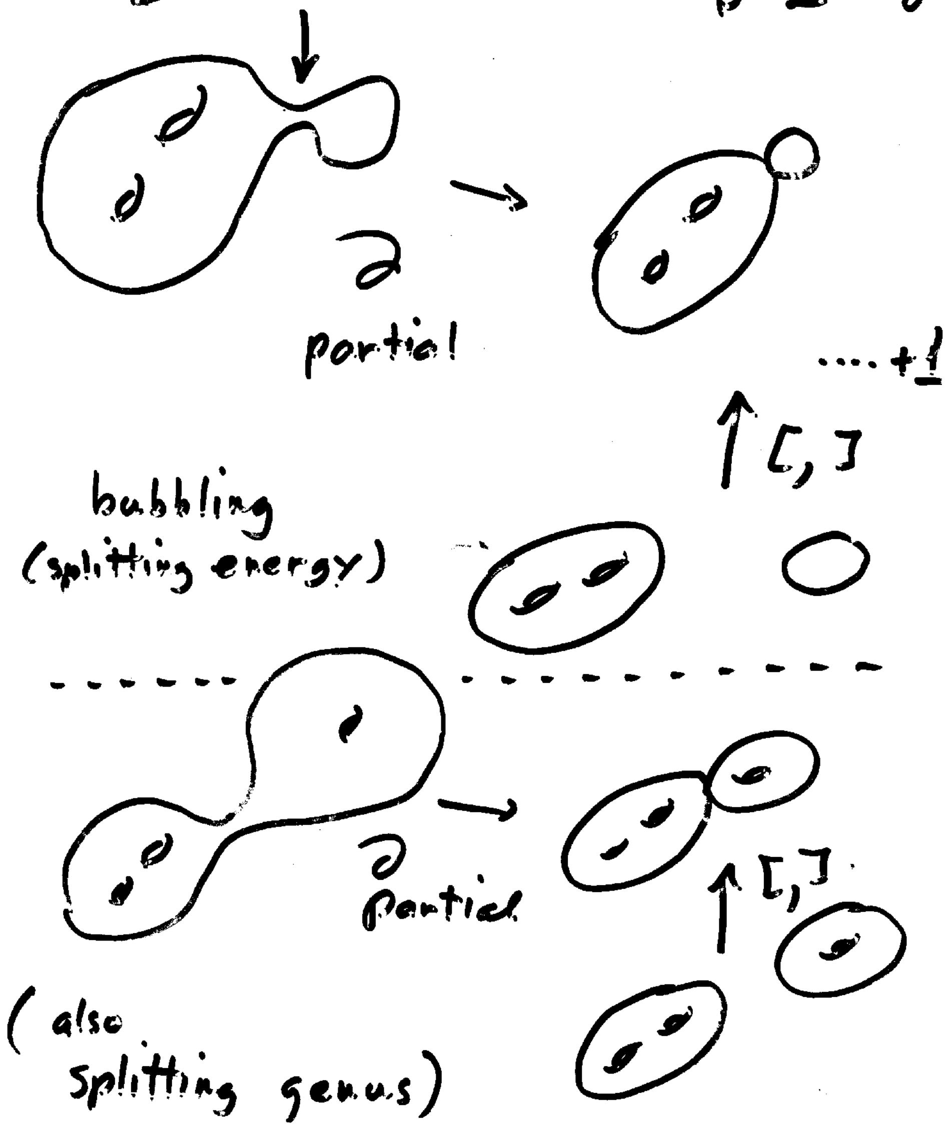
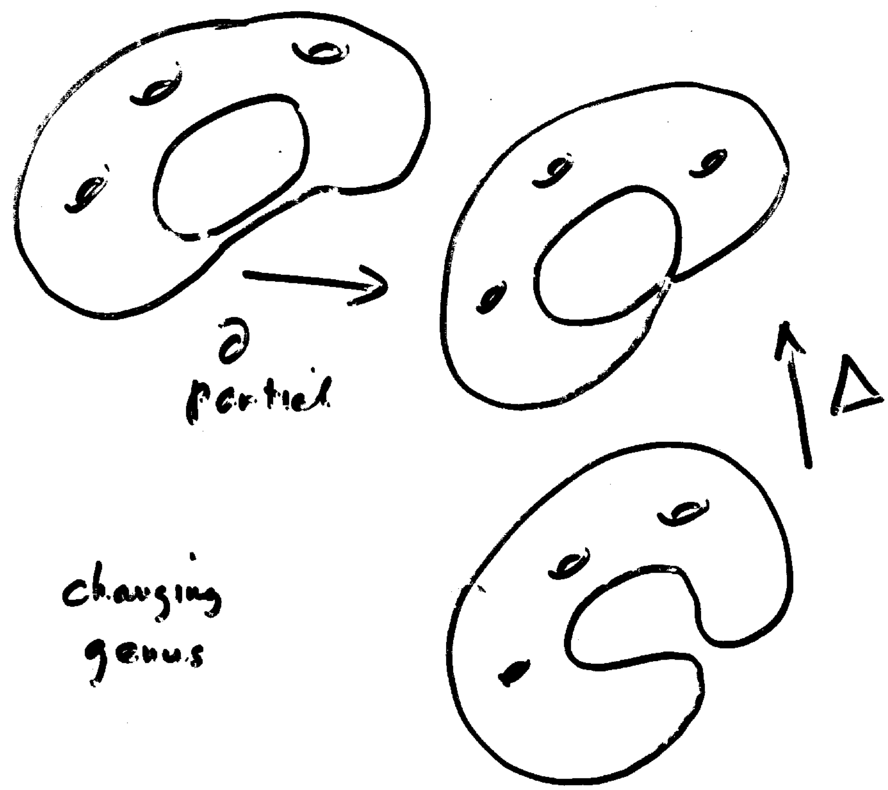

∂
partial
.... + 1
[,]
bubbling
(splitting energy)
∂
partial
[,]
(also
splitting genus)



recall Δ' at chain level

$(\Delta' = \partial \pm \Delta)$ Gromov chain X

satisfies $\partial X = \Delta X + [X,X]$

$$\boxed{\text{ie} \quad \Delta' X + [X,X] = 0}$$

This motivates ① defining for each mfld the "compactified membrane homology" using the differential $\partial + \Delta = \Delta'$ instead of ∂. (a Lie algebra)

② defining for mfld which admits symplecture ... the total Gromov chain $e^S = \sum \frac{X^n}{n!}$ for all disconnected surfaces

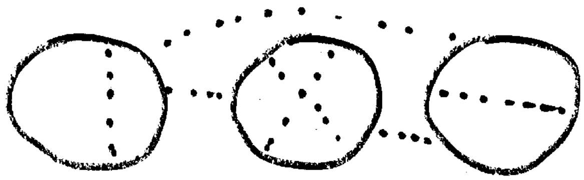
$\int e^S$

calculation $\Delta' X + [X, X] = 0$

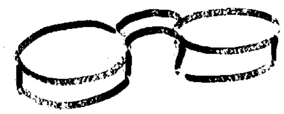
$$\Rightarrow \Delta' e^S = 0.$$

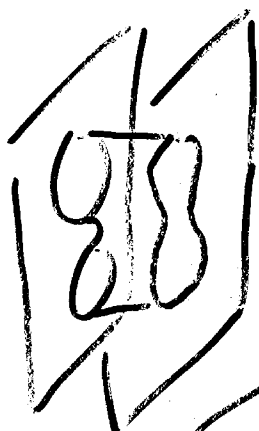
Thm $e^S \in H_{\langle membrane \rangle}$ (symplectic mfld) more string topology

closure families

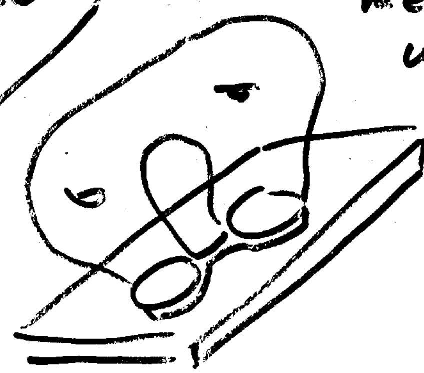


handle decompositions



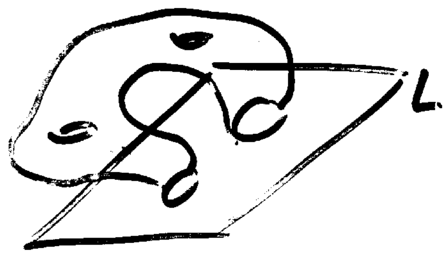


combine with membranes with ∂



Consider a manifold pair (M, L) or $(M, \partial M)$ and all mappings of surfaces $(\Sigma, \partial\Sigma) \longrightarrow (M, L)$ the two previous discussions come into play and we obtain a combined structure in the process of formulating

Again if M is (admits) symplectic structure and L is Lagrangian there is (again) the limited Gromov chain



and the structure of its ∂ uses the BV algebra and the Lie bialgebra operations mentioned above. The formalism fits with ∂M contact.