MATH 4200/6200: PROBLEM SET 1
DUE IN CLASS FRIDAY, AUGUST 29, 2008

1. Prove that, in any ordered field F (*i.e.*, any set F equipped with binary operations $+$ and $\cdot$ and an order relation $<$ that satisfy axioms 1-6 on pp. 30-31 of Munkres), one has

$$0 < 1.$$

2. Prove directly from the definition given in class (and possibly also from its immediate consequences, such as mathematical induction), the fact that if $m, k \in \mathbb{Z}_+$ and $k < m$ then $k + 1 \leq m$. (Since this fact was used in class and in the book to prove the well-ordering principle, you may not use the well-ordering principle in your proof.) Suggestion[1]: Show first that if $k, m \in \mathbb{Z}_+$ and $m > k$ then $m - k \in \mathbb{Z}_+$.

3 (Munkres p. 35, #8). (a) Show that $\mathbb{R}$ has the greatest lower bound property (*i.e.*, that if $A \subset \mathbb{R}$ is a nonempty subset with a lower bound then A has a greatest lower bound; this g.l.b. will be denoted $\inf A$).

(b) Show that $\inf\{1/n | n \in \mathbb{Z}_+\} = 0$.

(c) Show that, given a with $0 < a < 1$, $\inf\{a^n | n \in \mathbb{Z}_+\} = 0$. (Hint: Let $h = \frac{1-a}{a}$, and show that $(1 + h)^n \geq 1 + nh$.)

4 (Munkres p. 44, #1). (a) Make a list of all the injective maps

$$f : \{1, 2, 3\} \to \{1, 2, 3, 4\}.$$

Show that none of them is bijective. (This constitutes a *direct* proof that a set A of cardinality three does not have cardinality four.)

(b) How many injective maps

$$f : \{1, \ldots, 8\} \to \{1, \ldots, 10\}$$

are there? (You can see why one would not wish to prove directly that there is no bijective correspondence between these sets.)

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[1] This is an improvement on the more complicated suggestion that was in the original version of the problem set; you can still feel free to do the problem in a different way.