

**MATH 4200/6200: PROBLEM SET 12**  
**DUE IN CLASS FRIDAY, DECEMBER 5, 2008**

A. Munkres Section 33: 3, 9; Section 34: 1, 3, 4; Section 35: 4, 5, 6, 7

B. (a) Prove that any  $T_1$  space  $X$  which satisfies the conclusion of Urysohn's lemma (*i.e.*, for any disjoint closed  $A, B \subset X$  there is a continuous  $f: X \rightarrow [0, 1]$  such that  $f|_A = 0$  and  $f|_B = 1$ ) is normal.

(b) Give an example of a locally compact Hausdorff space  $X$  which is not normal.

(c) Although (a) and (b) together show that there are locally compact Hausdorff spaces for which Urysohn's lemma fails, prove the following restricted version: If  $X$  is a locally compact Hausdorff space,  $K \subset X$  is *compact*, and  $A \subset X$  is a closed subset with  $K \cap A = \emptyset$ , prove that there is a continuous function  $f: X \rightarrow \mathbb{R}$  such that  $f|_K = 1$  and  $f|_A = 0$ .

C. Extra credit: Munkres 33.11