MATH 4200/6200: MIDTERM
FRIDAY, OCTOBER 10, 2008

There are 100 points possible on the exam; the point value for each part is indicated in parentheses.

1 (20). Let $\mathcal{S}$ denote the collection of all subsets of $\mathbb{R}^2$ having form either $(-\infty, a) \times (-\infty, b)$ (where $a, b \in \mathbb{R}$) or $(c, \infty) \times (d, \infty)$ (where $c, d \in \mathbb{R}$). Prove that $\mathcal{S}$ is a subbasis for the standard topology on $\mathbb{R}^2$.

2. Consider the sequence $\{\underline{a}_n\}_{n=1}^{\infty}$ in $\mathbb{R}^{\mathbb{Z}_+}$ given by

$$(\underline{a}_n)_k = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

Define $\underline{0} \in \mathbb{R}^{\mathbb{Z}_+}$ to be the sequence all of whose terms are 0. Determine whether $\{\underline{a}_n\}_{n=1}^{\infty}$ converges to $\underline{0}$ in:
(a) (10) the box topology;
(b) (10) the product topology;
(c) (10) the uniform topology.

3 (20). Let $\{X_\alpha | \alpha \in J\}$ be a collection of topological spaces, let $\beta \in J$, and let $\pi_\beta : \prod_{\alpha \in J} X_\alpha \to X_\beta$ be the projection (sending $\{x_\alpha\}_{\alpha \in J}$ to x_β). Put the product topology on $\prod_{\alpha \in J} X_\alpha$. Prove that π_β is a quotient map.

4. (a) (15) Find a topology $\mathcal{T}$ on $\mathbb{Z}_+$ which is countably infinite (*i.e.*, the collection of open sets is countably infinite).
(b) (15) Find a topology $\mathcal{T}$ on $\mathbb{Z}_+$ which is Hausdorff but which is not equal to the discrete topology (Suggestion: First find one on some other countable set).