Math 2250 Exam 2 October 11, 2006

Name: Solutions (100 points total)

1. (38 points) Find y' for the following functions. **Do not simplify.**

(a) $y = \dfrac{\tan x}{x + x^{-1}}$

$$y' = \frac{(x+x^{-1})(\tan x)' - (\tan x)(x+x^{-1})'}{(x+x^{-1})^2}$$

$$= \frac{(x+x^{-1})\sec^2 x - \tan x\,(1 - x^{-2})}{(x+x^{-1})^2}$$

(b) $y = \sin^3(\sqrt{x}) = \left(\sin(x^{1/2})\right)^3$

$$y' = 3\left(\sin(x^{1/2})\right)^2 \cdot \left(\sin(x^{1/2})\right)'$$

$$= 3\left(\sin(x^{1/2})\right)^2 \cdot \cos(x^{1/2}) \cdot (x^{1/2})'$$

$$= 3\left(\sin(x^{1/2})\right)^2 \cdot \cos(x^{1/2}) \cdot \tfrac{1}{2}x^{-1/2}$$

(c) $y = (1+2x)^{3x}$

$$\ln y = \ln(1+2x)^{3x}$$

$$\ln y = 3x\ln(1+2x)$$

$$\cancel{\frac{1}{y} = y}$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(3x\ln(1+2x)\right)$$

$$\frac{1}{y}\cdot y' = (3x)'\ln(1+2x) + (3x)\ln(1+2x)'$$

$$\frac{1}{y}\cdot y' = 3\ln(1+2x) + 3x\cdot\frac{1}{1+2x}\cdot(1+2x)'$$

$$\frac{1}{y}\cdot y' = 3\ln(1+2x) + 3x\cdot\frac{1}{1+2x}\cdot 2$$

$$y' = y\left(3\ln(1+2x) + \frac{6x}{1+2x}\right)$$

$$= (1+2x)^{3x}\left(3\ln(1+2x) + \frac{6x}{1+2x}\right)$$

(d) $y = 3x^2 \sec(x^4 + 5x)$

$$y' = (3x^2)' \sec(x^4+5x) + (3x^2) \sec(x^4+5x)'$$

$$= 6x \sec(x^4+5x) + 3x^2 \cdot \sec(x^4+5x) \tan(x^4+5x) \cdot (x^4+5x)'$$

$$= 6x \sec(x^4+5x) + 3x^2 \cdot \sec(x^4+5x) \cdot \tan(x^4+5x) \cdot (4x^3+5)$$

2. (15 points) Consider the curve defined by $y^2 = x \cos y + 1$. Find y'.

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x\cos y + 1)$$

$$2y \cdot y' = x' \cos y + x(\cos y)' + 0$$

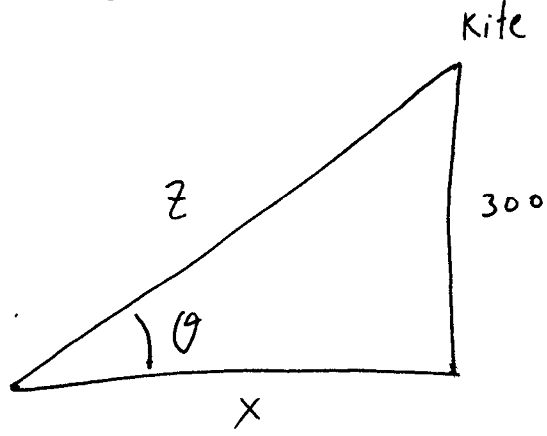
$$2y \cdot y' = 1 \cdot \cos y + -x \sin y \cdot y'$$

$$2y \cdot y' + x \sin y \cdot y' = 1 \cdot \cos y$$

$$y'(2y + x\sin y) = \cos y$$

$$y' = \frac{\cos y}{2y + x \sin y}$$

3. (26 points) A kite 300 ft above the ground moves horizontally at a speed of 8 ft/sec. At what rate is the angle between the string and the horizontal decreasing when 500 ft of string have been let out?



Given: $\frac{dx}{dt} = 8$

Find: $\frac{d\theta}{dt}$ when $z = 500$

$$\frac{x}{300} = \cot\theta$$

$$\frac{d}{dt}\left(\frac{x}{300}\right) = \frac{d}{dt}\left(\cot\theta\right)$$

$$\frac{1}{300}\cdot\frac{dx}{dt} = -\csc^2\theta\cdot\frac{d\theta}{dt}$$

when $z = 500$,

$$\csc\theta = \frac{1}{\sin\theta} = \frac{500}{300} = \frac{5}{3}$$

$$\frac{1}{300}\cdot 8 = -\left(\frac{5}{3}\right)^2\cdot\frac{d\theta}{dt}$$

$$\frac{8}{300} = \frac{-25}{9}\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{8}{300}\cdot\frac{9}{25} = \frac{-6}{625}\ \frac{\text{radians}}{\text{sec.}}$$

4. (11 points) For $t \geq 0$ a particle moves along a straight line with position $s(t) = -t^3 + 3t^2 + 9t$. At what time $t > 0$ does the particle change direction?

$$v(t) = s'(t) = -3t^2 + 6t + 9 = -3(t^2 - 2t - 3)$$
$$= -3(t-3)(t+1)$$

$v(t) = 0$ when $t = 3$, $\cancel{t = -1}$ ← want $t > 0$

Note that $v(t) > 0$ for $0 < t < 3$
$v(t) < 0$ for $3 < t$

$\therefore$ particle changes direction at $t = 3$.

5. (10 points) Find $f'(3)$ if $f(x) = (xg(x) - 2)^4$, and $g(3) = 4$, $g'(3) = -2$.

$$f'(x) = 4(xg(x) - 2)^3 \cdot (xg(x) - 2)'$$
$$= 4(xg(x) - 2)^3 \cdot (x' g(x) + x g'(x) - 0)$$
$$= 4(xg(x) - 2)^3 \cdot (g(x) + x g'(x))$$

$$\therefore f'(3) = 4(3g(3) - 2)^3 \cdot (g(3) + 3 \cdot g'(3))$$
$$= 4(3 \cdot 4 - 2)^3 \cdot (4 + 3 \cdot (-2))$$
$$= 4(10)^3 \cdot (-2) = -8000$$