Math 2250 Exam 3 November 13, 2006

Name: Solutions (100 points total)

1. (11 points) Use Newton's method to estimate the one real solution x of the equation $x^3 = -3x - 1$. Using initial estimate $x_0 = 0$, find x_2.

Find roots of: $f(x) = x^3 + 3x + 1$

$f'(x) = 3x^2 + 3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{(0)^3 + 3(0) + 1}{3(0)^2 + 3} = \frac{-1}{3}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{-1}{3} - \frac{\left(\frac{-1}{3}\right)^3 + 3\left(\frac{-1}{3}\right) + 1}{3\left(\frac{-1}{3}\right)^2 + 3} = \frac{-1}{3} - \frac{\frac{-1}{27}}{\frac{1}{3} + 3} = \boxed{\frac{-29}{90}}$$

2. (14 points) Find $\lim_{x \to 0^+} (1 + 3x)^{2/x}$

First look at $\lim_{x \to 0^+} \ln(1+3x)^{2/x}$:

$$\lim_{x \to 0^+} \ln(1+3x)^{2/x}$$

$$= \lim_{x \to 0^+} \frac{2\ln(1+3x)}{x}$$

$$= \lim_{x \to 0^+} \frac{2(\ln(1+3x))'}{x'}$$

l'Hôpital's rule

$$= \lim_{x \to 0^+} \frac{2 \cdot \frac{1}{1+3x} \cdot 3}{1} = 6$$

$$\therefore \ln(1+3x)^{2/x} \xrightarrow[x \to 0^+]{} 6$$

$$\therefore e^{\ln(1+3x)^{2/x}} \xrightarrow[x \to 0^+]{} \boxed{e^6}$$

$$\parallel$$

$$(1+3x)^{2/x}$$

3. (38 points) Consider the function $f(x)$ given below, whose first and second derivatives have been computed:

$$f(x) = x^{2/3}(x-5), \qquad f'(x) = \frac{5}{3}x^{-1/3}(x-2), \qquad f''(x) = \frac{10}{9}x^{-4/3}(x+1).$$

(a) Find the domain and all zeros of $f(x)$.

Domain: $(-\infty, \infty)$

Zeros: $x = 0$,
$x = 5$

$$f(x) = \sqrt[3]{x^2}(x-5)$$

$$f'(x) = \frac{5(x-2)}{3\sqrt[3]{x}}$$

$$f''(x) = \frac{10(x+1)}{9\sqrt[3]{x^4}}$$

(b) Find the regions of increasing and decreasing and all relative minima and maxima (both x- and y-coordinates) using the first derivative test.

	$x<0$	$0<x<2$	$2<x$
f'	$\frac{-}{-}$ $+$	$\frac{-}{+}$ $-$	$\frac{+}{+}$ $+$
f	incr. ↗	decr. ↘	incr. ↗

local max:

$x = 2$

$y = \sqrt[3]{2^2}(2-5) = -3\sqrt[3]{4}$

$(2, -3\sqrt[3]{4})$

local min:

$x = 0$

$y = \sqrt[3]{0^2}(0-5) = 0$

$(0,0)$

(c) Find the regions of concave up and concave down and all inflection points (x- and y-coordinates). Recheck the relative minima and maxima using the second derivative test.

	$x<-1$	$-1<x<0$	$0<x$
f''	$-$	$+$	$+$
f	conc. down	conc. up	conc. up

inflection pt:

$x=-1$

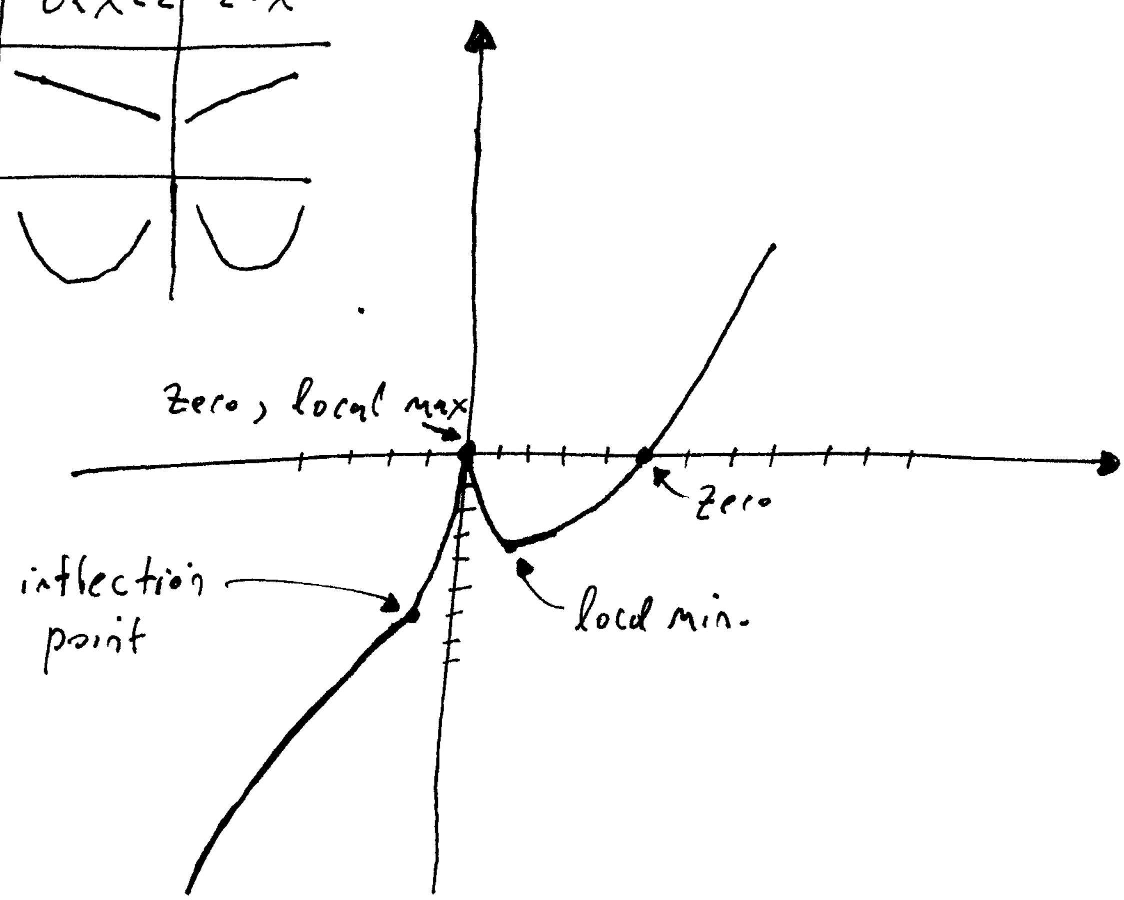
$y=\sqrt[3]{(-1)^2}(-1-5)=-6$

$(-1,-6)$

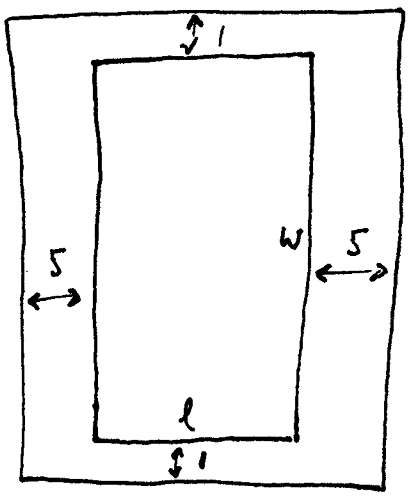
(Note: $f''(2)>0$ $\Rightarrow$ $x=2$ is a local min)

(d) Sketch the graph of $f(x)$, taking into account all the information determined in parts (a) - (c). Label the zeros, relative minima and maxima, and inflection points.

$x<-1$	$-1<x<0$	$0<x<2$	$2<x$



4. (25 points) You are designing a cardboard poster which must contain 80 in^2 of printing with a 1-in. margin on top and bottom and a 5-in. margin at each side. What overall dimensions for the poster will minimize the amount of cardboard used? (Note: you must show how you know that your answer gives you the minimum possible amount of cardboard).



Minimize:

$$A = (l+10)(w+2)$$
$$= lw + 10w + 2l + 20$$
$$= 80 + 10\left(\frac{80}{l}\right) + 2l + 20$$
$$= 100 + \frac{800}{l} + 2l$$

$$A' = \frac{-800}{l^2} + 2 = 0 \Rightarrow l^2 = \frac{800}{2} \Rightarrow l = \pm 20.$$

(take $l = +20$)

Constraints:

$w > 0$, $l > 0$,

$lw = 80$

$\left(w = \frac{80}{l}\right)$

$$A'' = \frac{1600}{l^3} > 0 \text{ for all } l > 0,$$

so $l = 20$ is a global minimum.

when $\boxed{l = 20}$, $\boxed{w = \frac{80}{20} = 4}$.

$\therefore$ Dimensions of poster: 30×6

5. Solve the initial value problem $\frac{dy}{dt} = 2 - 6t^2 + \sin\left(\frac{t}{4}\right)$, $y(0) = 5$. (12 points)

$$y = 2t - 2t^3 - 4\cos\left(\frac{t}{4}\right) + C$$

Use $y(0) = 5$ to find C

$$5 = 2(0) - 2(0)^3 - 4\cos\left(\frac{0}{4}\right) + C$$

$$5 = -4 + C \Rightarrow C = 9$$

$$\boxed{\therefore y = 2t - 2t^3 - 4\cos\left(\frac{t}{4}\right) + 9}$$