Math 2260 Exam 2 October 16, 2007

Name: Solutions (100 points total)

1. (70 points) Evaluate the following integrals.

(a) $\int \sin^4(2x)\,dx$

$$= \int \left(\sin^2(2x)\right)^2 dx$$

$$= \int \left(\frac{1-\cos(4x)}{2}\right)^2 dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos(4x) + \cos^2(4x)\right) dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos(4x) + \frac{1+\cos(8x)}{2}\right) dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos(4x) + \frac{1}{2} + \frac{1}{2}\cos(8x)\right) dx$$

$$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos(4x) + \frac{1}{2}\cos(8x)\right) dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x - \frac{1}{2}\overset{\sin}{\cancel{\cos}}(4x) + \frac{1}{16}\sin(8x)\right) + C$$

$$\text{(b)} \int \overbrace{(\ln(4x))^2}^{u}\, \overbrace{dx}^{dv} = \overbrace{(\ln(4x))^2}^{u} \cdot \overbrace{x}^{v} - \int \overbrace{x}^{v} \cdot \overbrace{2\ln(4x)\cdot\frac{1}{x}\,dx}^{du}$$

$$\boxed{\begin{array}{ll} u=(\ln(4x))^2 & v=x \\ du=2(\ln(4x))\cdot\frac{1}{x}\,dx & dv=dx \end{array}}$$

$$= (\ln(4x))^2 x - 2\Big[\underbrace{\int \ln(4x)\,dx}\Big]$$

$$\left(\begin{array}{l} \int \ln(4x)\,dx = \ln(4x)\cdot x - \int x\cdot\frac{1}{x}\,dx \\ \boxed{\begin{array}{ll} u=\ln(4x) & v=x \\ du=\frac{1}{x}\,dx & dv=dx \end{array}} = \ln(4x)\cdot x - x \end{array} \right)$$

$$= \ln(4x)^2\cdot x - 2\left[\ln(4x)\cdot x - x\right] + C$$

(c) $\int \frac{x^3+8x^2+25x+32}{x^2+6x+8}\,dx$

$$\begin{array}{r|l} & \quad x+2 \\ \hline x^2+6x+8 & x^3+8x^2+25x+32 \\ & x^3+6x^2+8x \\ \hline & \qquad 2x^2+17x+32 \\ & \qquad 2x^2+12x+16 \\ \hline & \qquad\quad 5x+16 \end{array}$$

$$\therefore \frac{x^3+8x^2+25x+32}{x^2+6x+8} = x+2+\frac{5x+16}{x^2+6x+8}$$

$$\frac{5x+16}{(x+4)(x+2)} = \frac{2}{x+4} + \frac{3}{x+2}$$

$$\therefore \int \frac{x^3+8x^2+25x+32}{x^2+6x+8}\,dx$$

$$= \int\left(x+2+\frac{2}{x+4}+\frac{3}{x+2}\right)dx$$

$$= \frac{1}{2}x^2+2x+2\ln|x+4|+3\ln|x+2|+C$$

(d) $\int \frac{dx}{x^2 + 4x + 13}$

$$= \int \frac{\overset{du}{\boxed{dx}}}{(\underset{u}{\boxed{x+2}})^2 + 9}$$

$$= \int \frac{du}{u^2 + 9}$$

$$= \int \frac{\frac{1}{9}\, du}{\frac{1}{9}(u^2 + 9)}$$

$$= \int \frac{\frac{1}{9}\, du}{\frac{u^2}{9} + 1}$$

$$= \frac{1}{9} \int \frac{\overset{3dz}{\boxed{du}}}{(\underset{z}{\boxed{\frac{u}{3}}})^2 + 1}$$

$$= \frac{1}{9} \int \frac{3\, dz}{z^2 + 1}$$

$$= \frac{1}{3} \int \frac{dz}{z^2 + 1}$$

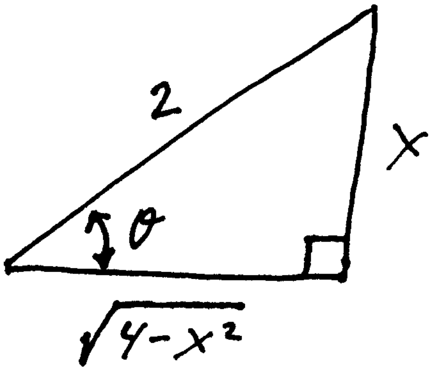
$$= \frac{1}{3} \tan^{-1}(z) + C$$

$$= \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) + C$$

$$= \frac{1}{3} \tan^{-1}\left(\frac{x+2}{3}\right) + C$$

2. (15 points) Which of the following integrals can $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$ be converted to using trigonometric substitution? Show your work.

(a) $\int 4\sin\theta\,d\theta$ ● $\int 4\sin^2\theta\,d\theta$ (c) $\int 4\tan\theta\,d\theta$ (d) $\int 4\tan^2\theta\,d\theta$
(e) $\int 4\sec\theta\,d\theta$ (f) $\int 4\sec^2\theta\,d\theta$ (g) $\int 4\,d\theta$ (h) $\int 4\theta\,d\theta$



$\sqrt{4-x^2} = 2\cos\theta$

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$$\therefore \int \frac{x^2}{\sqrt{4-x^2}}\,dx = \int \frac{(2\sin\theta)^2}{\cancel{2\cos\theta}} \cdot \cancel{2\cos\theta}\,d\theta$$

(b)

3. (15 points) Determine whether $\int_2^\infty \frac{1}{x^2}\,dx$ converges or diverges. If it converges, evaluate it. Justify your answer.

$$\int_2^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty}\left(\int_2^b \frac{1}{x^2}\,dx\right)$$

$$= \lim_{b\to\infty}\left(\frac{-1}{x}\Big|_2^b\right)$$

$$= \lim_{b\to\infty}\left(-\frac{1}{b} - \left(\frac{-1}{2}\right)\right)$$

$$= \lim_{b\to\infty}\left(\frac{1}{2} - \frac{1}{b}\right) = \frac{1}{2},$$

i.e., the improper integral converges to [redacted] $\frac{1}{2}$