

Name: _____

(100 points total)

1. (20 points) Determine whether the following series converge or diverge. Justify your answers and state all convergence tests used.

(a)
$$\sum_{n=3}^{\infty} \frac{n^2 + 5\sqrt{n} + 7}{n^4 - n^2 - 3}$$

(b)
$$\sum_{n=1}^{\infty} \left(\frac{1}{2} + \frac{1}{n^2} \right)$$

2. (20 points) Determine whether the following series converge or diverge. **If a series converges, find its sum.** Justify your answers and state all convergence tests used.

(a)
$$\sum_{n=2}^{\infty} \frac{4^n + 1}{3^{2n}}$$

(b)
$$\sum_{n=1}^{\infty} \left(\frac{n}{2n+3} - \frac{n+1}{2(n+1)+3} \right)$$

3. (20 points) Does $\sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln n}$ converge absolutely, converge conditionally, or diverge? Justify your answer, and state all convergence tests used.

4. (20 points) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{3^n \sqrt{n} (x-4)^n}{(n+2)!}$.

5. (20 points) Let $\sum_{i=1}^{\infty} a_i$ be a series. Define $s_n = \sum_{i=1}^n a_i$. Indicate whether each of the following statements is true or false (no explanations required).

- (a) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{i=n}^{\infty} a_n$ must converge.
- (b) If $\lim_{n \rightarrow \infty} s_n = 2$, then $\sum_{i=n}^{\infty} a_n$ must converge.
- (c) If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, then $\sum_{n=1}^{\infty} a_n$ must converge.
- (d) If $\sum_{n=1}^{\infty} |a_n|$ diverges, then $\sum_{n=1}^{\infty} a_n$ must diverge.