

Name: \_\_\_\_\_

(100 points total)

1. (40 points) Determine whether the following series converge or diverge. **If a series converges, find its sum.** Justify your answers and state all convergence tests used.

(a)  $\sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{1}{4} + \frac{1}{n^2}}$ .

(d)  $\sum_{k=1}^{\infty} a_k$ , if the partial sum  $s_n = \sum_{k=1}^n a_k$  equals  $\frac{4n^2}{n^2 + 3}$ .

(c)  $\sum_{n=0}^{\infty} \frac{3^n}{n!}$

(b)  $\sum_{n=2}^{\infty} (-1)^n \frac{5}{3^{2n}}$

2. (15 points) Does  $\sum_{n=3}^{\infty} (-1)^n \left( \frac{1 + \cos n}{n^2} \right)$  converge absolutely, converge conditionally, or diverge? State all convergence tests used.

3. (30 points) Find the radius **and** interval of convergence of  $\sum_{n=2}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x-3)^n$ .

3. (15 points) Find the third order Taylor polynomial of  $f(x) = \sqrt{x}$  at  $a = 1$ .