Math 2260 Quiz 10 April 24, 2007

Name: <u>Solutions</u> (20 points total)

1. (7 points) Let $\mathbf{u} = 5\mathbf{i} - 12\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Find
(a) The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.
(b) $\text{proj}_{\mathbf{v}}\mathbf{u}$.

**Solution**.

$$\begin{aligned}
\mathbf{u} \bullet \mathbf{v} &= 5 \cdot 2 + 0 \cdot 2 + (-12) \cdot 1 = -2 \\
|\mathbf{u}| &= \sqrt{5^2 + 0^2 + (-12)^2} = 13 \\
|\mathbf{v}| &= \sqrt{2^2 + 2^2 + (1)^2} = 3
\end{aligned}$$

(a) We learned that $\mathbf{u} \bullet \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$. Substituting the above values: $-2 = (13)(3)\cos\theta$, so $\cos\theta = -2/39$.

(b) $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{v}|^2}\right) \mathbf{v} = (-2/9)(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = (-4/9)\mathbf{i} + (-4/9)\mathbf{j} + (-2/9)\mathbf{k}$.

2. (5 points) Find a vector of magnitude 8 in the direction of $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. Write your answer in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

**Solution**. $8\,\dfrac{\mathbf{u}}{|\mathbf{u}|} = 8\,\dfrac{\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}}{\sqrt{1^2 + 2^2 + (-2)^2}} = (8/3)\mathbf{i} + (16/3)\mathbf{j} + (-16/3)\mathbf{k}$.

3. (5 points) Using either a single equation or a pair of equations, describe the circle of radius 3 centered at the point $(4, 0, 0)$ and lying in the plane $x = 4$.

**Solution**. $y^2 + z^2 = 9$, $x = 4$. (The graph of $y^2 + z^2 = 9$ is a cylinder of radius 3 with the $x$-axis passing through its center. The graph of $x = 4$ is a plane parallel to the $yz$-plane, passing through the point $(4, 0, 0)$. The intersection of the cylinder and plane is the desired circle.)

4. (3 points) Are the vectors $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ perpendicular? Justify your answer.

**Solution**. $\mathbf{u} \bullet \mathbf{v} = 3 \cdot (-3) + 4 \cdot 3 + (-1) \cdot 2 = 1$. Since this dot product is not 0, $\mathbf{u}$ and $\mathbf{v}$ are not perpendicular.