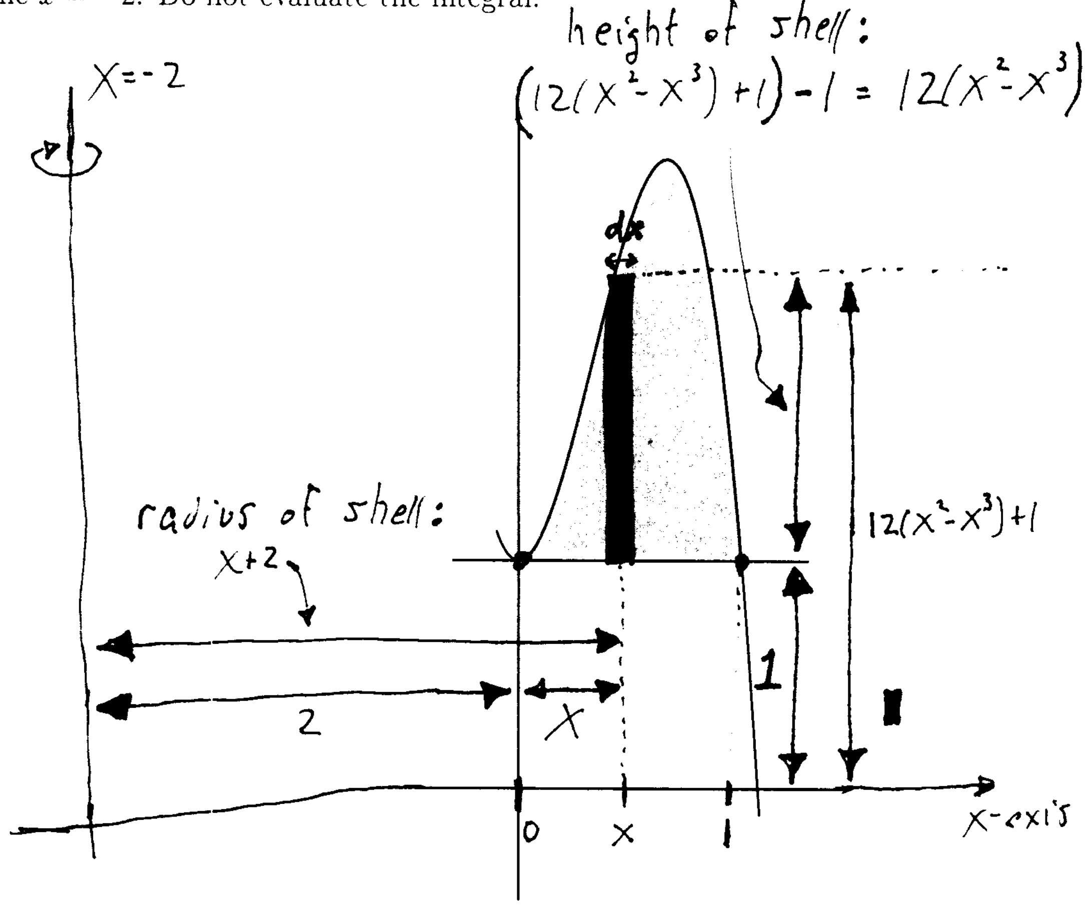
Math 2260 Quiz 2 January 30, 2007

Name: Solutions (20 points total)

1. (10 points) The region R which lies in the first quadrant below the curve $y = 12(x^2 - x^3) + 1$ and above the line $y = 1$ is sketched below. Give a definite integral which computes the volume of the solid generated by revolving R about the line $x = -2$. Do not evaluate the integral.

$$12(x^2 - x^3) + 1 = 1$$
$$12(x^2 - x^3) = 0$$
$$12x^2(1 - x) = 0$$
$$x = 0 \text{ or } x = 1$$



$$\text{Volume} = \int_0^1 2\pi \begin{pmatrix}\text{radius of}\\ \text{shell}\end{pmatrix} \cdot \begin{pmatrix}\text{height of}\\ \text{shell}\end{pmatrix} \cdot dx$$

$$\boxed{= \int_0^1 2\pi (x+2) \cdot 12(x^2 - x^3)\, dx}$$

2. (10 points) Find the length of the curve $x = (y^4/4) + 1/(8y^2)$, $1 \le y \le 2$. (Hint: $1 + (dx/dy)^2$ is a perfect square.)

$$x = \frac{1}{4}y^4 + \frac{1}{8}y^{-2}$$

$$\frac{dx}{dy} = \frac{1}{4}\cdot 4y^3 + \frac{1}{8}\cdot -2y^{-3}$$

$$= y^3 - \frac{1}{4}y^{-3}$$

$$\text{length} = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= \int_1^2 \sqrt{1+\left(y^3 - \frac{1}{4}y^{-3}\right)^2}\,dy$$

$$= \int_1^2 \sqrt{1+\left(y^6 - \frac{1}{2} + \frac{1}{16}y^{-6}\right)}\,dy$$

$$= \int_1^2 \sqrt{y^6 + \frac{1}{2} + \frac{1}{16}y^{-6}}\,dy$$

$$= \int_1^2 \sqrt{\left(y^3 + \frac{1}{4}y^{-3}\right)^2}\,dy$$

$$= \int_1^2 \left(y^3 + \frac{1}{4}y^{-3}\right)dy$$

$$= \left(\frac{y^4}{4} + \frac{1}{4}\cdot\frac{y^{-2}}{-2}\right)\Bigg|_1^2 = \left(\frac{(2)^4}{4} + \frac{(2)^{-2}}{-8}\right) - \left(\frac{(1)^4}{4} + \frac{(1)^{-2}}{-8}\right)$$

$$\boxed{= \frac{123}{32}}$$