Math 2260 Quiz 4 February 20, 2007

Name: Solutions (20 points total)

1. (10 points) Find the y-coordinate of the center of mass of a thin plate of constant density δ covering the region between the curve $y = x^2$ and the x-axis from $x = 1$ to $x = 2$.

$$M_x = \int_1^2 \frac{1}{2}\delta (x^2)^2 dx = \frac{1}{2}\delta \int_1^2 x^4 dx = \frac{1}{2}\delta \cdot \frac{x^5}{5}\Big|_1^2$$

$$= \frac{1}{2}\delta\left(\frac{32}{5} - \frac{1}{5}\right) = \frac{31\delta}{10}$$

$$m = \int_1^2 \delta x^2 dx = \delta \frac{x^3}{3}\Big|_1^2 = \delta \frac{8-1}{3} = \frac{7\delta}{3}$$

$$\bar{y} = \frac{M_x}{m} = \frac{\frac{31\cancel{\delta}}{10}}{\frac{7\cancel{\delta}}{3}} = \boxed{\frac{93}{70}}$$

$\int u\,dv = uv - \int v\,du$

2. (10 points) Evaluate $\int \sqrt[3]{x} \ln x \, dx$.

$$\int \overset{u}{(\ln x)} \overset{dv}{(x^{1/3}\,dx)} = (\ln x)\cdot\left(\frac{3}{4}x^{4/3}\right) - \int \frac{3}{4}x^{4/3}\cdot\frac{1}{x}\,dx$$

$$\left(\begin{array}{ll} u = \ln x & v = \frac{3}{4}x^{4/3} \\ du = \frac{1}{x}dx & dv = x^{1/3}dx \end{array}\right)$$

$$= (\ln x)\cdot\left(\frac{3}{4}x^{4/3}\right) - \frac{3}{4}\int x^{4/3}\cdot x^{-1}\,dx$$

$$= (\ln x)\cdot\left(\frac{3}{4}x^{4/3}\right) - \frac{3}{4}\int x^{1/3}\,dx$$

$$= (\ln x)\cdot\left(\frac{3}{4}x^{4/3}\right) - \frac{3}{4}\cdot\frac{3}{4}x^{4/3} + C$$

$$\boxed{= (\ln x)\cdot\left(\frac{3}{4}x^{4/3}\right) - \frac{9}{16}x^{4/3} + C}$$