Math 2260 Quiz 8 April 3, 2007

Name: Solutions (20 points total)

1. (20 points) Determine whether each of the following series converges or diverges. If a series converges, find its sum if possible. State all convergence tests used.

(a) $\sum_{n=3}^{\infty} \frac{1}{n \ln n}$

$$\int_3^\infty \frac{1}{x \ln x}\,dx = \int_3^\infty \underbrace{\frac{1}{\ln x}}_{u} \cdot \underbrace{\frac{1}{x}\,dx}_{du} = \ln(\overset{u}{\ln x})\Big|_3^\infty$$

$$= \ln(\ln(\infty)) - \ln(\ln(3)) = \infty$$

so integral diverges, which implies that series $\boxed{\text{diverges}}$ (Integral Test).

(b) $\sum_{n=1}^{\infty} \frac{n}{3n+1}$

$$\lim_{n \to \infty} \frac{n}{3n+1} = \frac{1}{3} \neq 0$$

$\therefore$ Series $\boxed{\text{diverges}}$ by n-th term test for divergence.

(c) $\sum_{n=2}^{\infty} 5(-1)^n \, (3/2)^{-2n}$

$$= \sum_{n=2}^{\infty} 5(-1)^n\left(\left(\frac{3}{2}\right)^{-2}\right)^n = \sum_{n=2}^{\infty} 5(-1)^n \cdot \left(\frac{1}{\left(\frac{3}{2}\right)^2}\right)^n$$

$$= \sum_{n=2}^{\infty} 5(-1)^n\left(\frac{4}{9}\right)^n = \sum_{n=2}^{\infty} 5\left((-1)\cdot\left(\frac{4}{9}\right)\right)^n$$

$$= \sum_{n=2}^{\infty} 5\left(\frac{-4}{9}\right)^n.$$ Geometric series, $r = \frac{-4}{9}$. $|r| = \frac{4}{9} < 1 \Rightarrow \boxed{\text{Converges}}$

Converges to $\boxed{\dfrac{5\left(-\frac{4}{9}\right)^2}{1-\left(-\frac{4}{9}\right)}}$

(d) $\sum_{n=3}^{\infty} \frac{5}{n\sqrt[3]{n}} = \sum_{n=3}^{\infty} \frac{5}{n^{4/3}}$

$\boxed{\text{Converges}}$, p-series, $p = \frac{4}{3} (> 1)$.

(e) $\sum_{n=1}^{\infty} \left(\frac{3}{\sqrt{n}} - \frac{3}{\sqrt{n+1}} \right)$ Telescoping Series.

$S_1 = \frac{3}{1} - \frac{3}{\sqrt{2}}$

$S_2 = \left(\frac{3}{1} - \cancel{\frac{3}{\sqrt{2}}}\right) + \left(\cancel{\frac{3}{\sqrt{2}}} - \frac{3}{\sqrt{3}}\right) = \frac{3}{1} - \frac{3}{\sqrt{3}}$

$\vdots$

$S_n = \left(\frac{3}{1} - \cancel{\frac{3}{\sqrt{2}}}\right) + \left(\cancel{\frac{3}{\sqrt{2}}} - \cancel{\frac{3}{\sqrt{3}}}\right) + \left(\cancel{\frac{3}{\sqrt{3}}} - \cancel{\frac{3}{\sqrt{4}}}\right) + \cdots + \left(\cancel{\frac{3}{\sqrt{n}}} - \frac{3}{\sqrt{n+1}}\right) = \frac{3}{1} - \frac{3}{\sqrt{n+1}}$

$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(\frac{3}{1} - \frac{3}{\sqrt{n+1}} \right) = \frac{3}{1} = 3$

$\therefore$ Series $\boxed{\text{converges}}$ to 3.

By definition of convergence of a series.