

Name: _____

(100 points total)

1. (12 points) Find $\sin(-\pi/12)$.

2. (50 points) Determine and state whether each of the following statements is True or False. If True, then prove. If False, then **give a counterexample**.

(a) Let R be a commutative ring. If $f(x) \in R[x]$ and $\deg(f(x)) = d$, where $d \geq 1$, then $f(x)$ has at most d distinct roots in R .

(b) $x^3 + 2x + 1 \in \mathbb{Z}_3[x]$ is irreducible.

(c) Let R be a subring of S . If S is commutative, then R is commutative (note: it is not enough to say that a property is or is not inherited – you must prove it).

(d) Let $\alpha, \beta \in \mathbb{C}$. If $\mathbb{Q}[\alpha] \subset \mathbb{Q}[\beta]$, then $\mathbb{Q}[\alpha + \beta] \subset \mathbb{Q}[\beta]$.

3. (20 points) Prove that if p is a prime number, then \sqrt{p} is irrational.

4. (18 points) Find $(1/6)^{20} (\sqrt{3} + i\sqrt{3})^{40}$. Your answer should be in the form $a + bi$, where a and b are real numbers.