

Name: \_\_\_\_\_

(100 points total)

1. (25 points) Let  $f(x) = \frac{5}{x^2}$ .

(a) Using the **definition of derivative**, find the derivative of  $f(x)$ . Your answer should be a function of  $x$ .

(b) Give an equation of the tangent line to the graph of  $f(x)$  at  $x = 2$ .

2. (23 points) Find the following limits.

(a)  $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 + x - 12}$

(b)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 12}{x^3 - 3x^2 + 5x + 1}$

(c)  $\lim_{x \rightarrow \infty} \frac{\cos 5x}{2x}$

(d)  $\lim_{x \rightarrow 0} \frac{x^3 - 2x + \sin x}{4x}$

3. (20 points) Consider

$$f(x) = \begin{cases} 9 - x^2, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ x^4 + 4x + 3, & \text{if } x > 1 \end{cases}$$

(a) Is  $f(x)$  defined at  $x = 1$ ? If so, what is  $f(1)$ ?

(b) Determine whether  $\lim_{x \rightarrow 1^-} f(x)$  and  $\lim_{x \rightarrow 1^+} f(x)$  exist, and if so compute them.

(c) Does  $\lim_{x \rightarrow 1} f(x)$  exist? Justify your answer. If the limit does exist, then what is the limit?

(d) Is  $f(x)$  continuous at  $x = 1$ ? Justify your answer.

4. (25 points) Consider  $f(x) = \frac{-3x^2}{x^2 - 4}$ .

(a) Find the equations of all horizontal and vertical asymptotes of  $f(x)$ .

(b) Compute the left and right hand limits at each vertical asymptote.

*continuation of problem 4*

(c) Sketch the graph of  $f(x)$ . Your graph should incorporate all of the information you determined in parts (a) and (b).

5. (7 points) Let  $g(x)$  be a function. In one English sentence, explain the geometrical meaning of the following mathematical statement:  $g'(7) = 5$ . The only mathematical symbols your sentence is allowed to contain are ' $g(x)$ ', ' $x$ ', ' $7$ ', and ' $5$ '. Your sentence should be grammatically correct.