Math 2250 Exam 2 March 6, 2008

Name: Solutions (100 points total)

1. (18 points) Find an equation of the tangent line to the curve $y^3 = 8 + 5xy^2 - \sin x$ at the point $(0, 2)$.

$$\frac{d}{dx}\left(y^3\right) = \frac{d}{dx}\left(8 + 5xy^2 - \sin x\right)$$

$$3y^2 \cdot \frac{dy}{dx} = 0 + \frac{d}{dx}\left(5xy^2\right) - \cos x$$

$$3y^2 \cdot \frac{dy}{dx} = 0 + \left(\frac{d}{dx}5x\right)\left(y^2\right) + \left(5x\right)\left(\frac{d}{dx}y^2\right) - \cos x$$

$$3y^2 \cdot \frac{dy}{dx} = 5 \cdot y^2 + 5x \cdot 2y\,\frac{dy}{dx} - \cos x$$

$$\left(3y^2 - 10xy\right)\frac{dy}{dx} = 5y^2 - \cos x$$

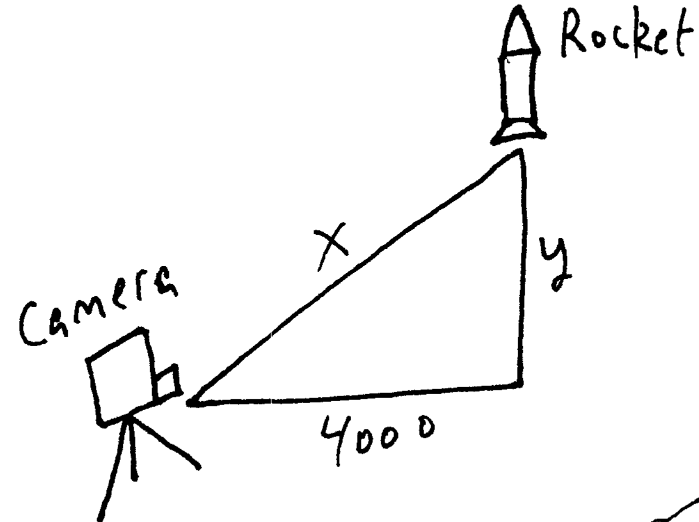
$$\frac{dy}{dx} = \frac{5y^2 - \cos x}{3y^2 - 10xy}$$

slope of tangent line $= \frac{dy}{dx} @ (0,2) = \frac{5(2)^2 - \cos(0)}{3(2)^2 - 10 \cdot 0 \cdot 2} = \frac{19}{12}$

Point on tangent line : $(0,2)$

$\therefore$ Eqn. of tangent line : $\boxed{y = 2 + \frac{19}{12}(x - 0)}$

2. (18 points) A television camera is positioned 4000 ft from the base of a rocket launching pad. A rocket rises vertically and its speed is 400 ft/sec when it has risen 3000 ft. How fast is the distance from the television camera to the rocket changing at that moment?



Given: $\frac{dy}{dt} = 400$ when $y = 3000$

Find: $\frac{dx}{dt}$ at this time

Note: when $y = 3000$,

$$x = \sqrt{(4000)^2 + (3000)^2} = 5000$$

$$\cancel{x^2}\ (4000)^2 + y^2 = x^2$$

$$\frac{d}{dt}\left((4000)^2 + y^2\right) = \frac{d}{dt}\left(x^2\right)$$

$$0 + 2y \cdot \frac{dy}{dt} = 2x \cdot \frac{dx}{dt}$$

$$2 \cdot 3000 \cdot 400 = 2 \cdot 5000 \cdot \frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = 240 \text{ ft/sec.}$$

3. (42 points) Find y' for the following functions. **Do not simplify**.

(a) $y = (\ln x)^{\tan x}$

$$\ln(y) = \ln\left((\ln x)^{\tan x}\right)$$

$$\ln y = \tan x \cdot \ln(\ln x)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\tan x \cdot \ln(\ln x)\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sec^2 x \cdot \ln(\ln x) + \tan x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = (\ln x)^{\tan x}\left[\sec^2 x \cdot \ln(\ln x) + \tan x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}\right]$$

(b) $y = (5x^2)(\tan^{-1}(7x^2 + 5))$

$$y' = (5x^2)'(\tan^{-1}(7x^2+5)) + (5x^2)(\tan^{-1}(7x^2+5))'$$

$$= 10x \cdot \tan^{-1}(7x^2+5) + 5x^2 \cdot \frac{1}{1+(7x^2+5)^2} \cdot 14x$$

(c) $y = \dfrac{\sqrt[5]{x} + x}{e^x + x^2}$ $= \dfrac{x^{1/5} + x}{e^x + x^2}$

$$y' = \frac{(x^{1/5}+x)'(e^x+x^2) - (x^{1/5}+x)(e^x+x^2)'}{(e^x+x^2)^2}$$

$$= \frac{(\frac{1}{5}x^{-4/5}+1)(e^x+x^2) - (x^{1/5}+x)(e^x+2x)}{(e^x+x^2)^2}$$

(d) $y = \sin^5(x\cos(x))$ $= [\sin(x\cos x)]^5$

$$y' = 5[\sin(x\cos x)]^4 \cdot (\sin(x\cos x))'$$

$$= 5[\sin(x\cos x)]^4 \cdot \cos(x\cos x) \cdot (x\cos x)'$$

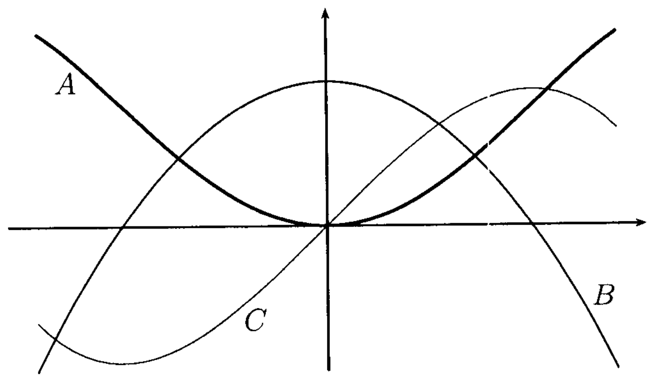
$$= 5[\sin(x\cos x)]^4 \cdot \cos(x\cos x) \cdot (1 \cdot \cos x - x\sin x)$$

4. (10 points) Identify the graphs A, B, and C as the graphs of a function and its derivatives:

A is the graph of the function

C is the graph of the function's first derivative

B is the graph of the function's second derivative



meter

5. (12 points) A rock shot vertically upward from the roof of a 180 ~~feet~~ building on the planet Tatooine reaches a height of $s(t) = -3t^2 + 12t + 180$ meters in t seconds. What is the velocity of the rock when it strikes the surface of the planet?

$s(t) = -3t^2 + 12t + 180$

$v(t) = s'(t) = -6t + 12$

- Find time when rock strikes planet.

$$-3t^2 + 12t + 180 = 0$$

$$-3(t^2 - 4t - 60) = 0$$

$$-3(t-10)(t+6) = 0$$

$t = 10$ or $t = \cancel{-6}$ ← want positive time

- Find velocity @ $t=0$.

$$v(10) = -6(10) + 12 = -48 \ \frac{m}{sec.}$$