Do not write in the boxes immediately below.

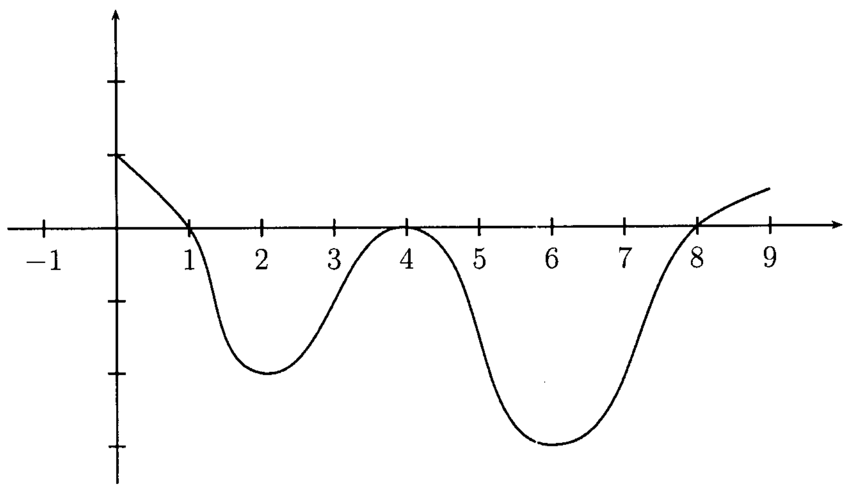
Question:	1	2	3	4	5	Total
Points:	10	30	28	8	24	100
Score:						

MATH 2250 Exam 3

April 3, 2008

Name: Solutions

1. (10 points) The figure below shows the graph of **the derivative** $f'(x)$ of a function defined on $(0, 9)$.



(a) Give all values of x in $(0, 9)$ where the function $f(x)$ has a local minimum.

$X = 8$

(b) Give all values of x in $(0, 9)$ where the function $f(x)$ has a local maximum.

$X = 1$

2. (30 points) Consider the function $f(x) = x^4 - 4x^3 + 10$.

(a) Find the regions of increasing and decreasing and all relative minimums and maximums (both x and y coordinates) of $f(x)$.

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

crit. points: $4x^2(x-3) = 0 \longleftrightarrow x=0$ or $x=3$

	$(-\infty,0)$	$(0,3)$	$(3,\infty)$
Sign of $f'(x)$	$-$	$-$	$+$
Behavior of $f(x)$	dec.	dec.	inc.

(relative) local min. @ $x=3$.

y-coor: $f(3) = 3^4 - 4\cdot 3^3 + 10$
$= 81 - 108 + 10$
$= -17$

$\therefore$ (relative) local min @ $(3,-17)$

(b) Find the regions of concave up and concave down and all inflection points (both x and y coordinates) of $f(x)$.

$f''(x) = 12x^2 - 24x = 12x(x-2)$

double crit. points: $12x(x-2) = 0 \longleftrightarrow x=0$ or $x=2$

	$(-\infty,0)$	$(0,2)$	$(2,\infty)$
Sign of $f''(x)$	$+$	$-$	$+$
Behavior of $f(x)$	cc-up	cc-down	cc-up

Infl. pt. @ $x=0$

y-coor: $f(0) = 0^4 - 4\cdot 0^3 + 10$
$= 10$

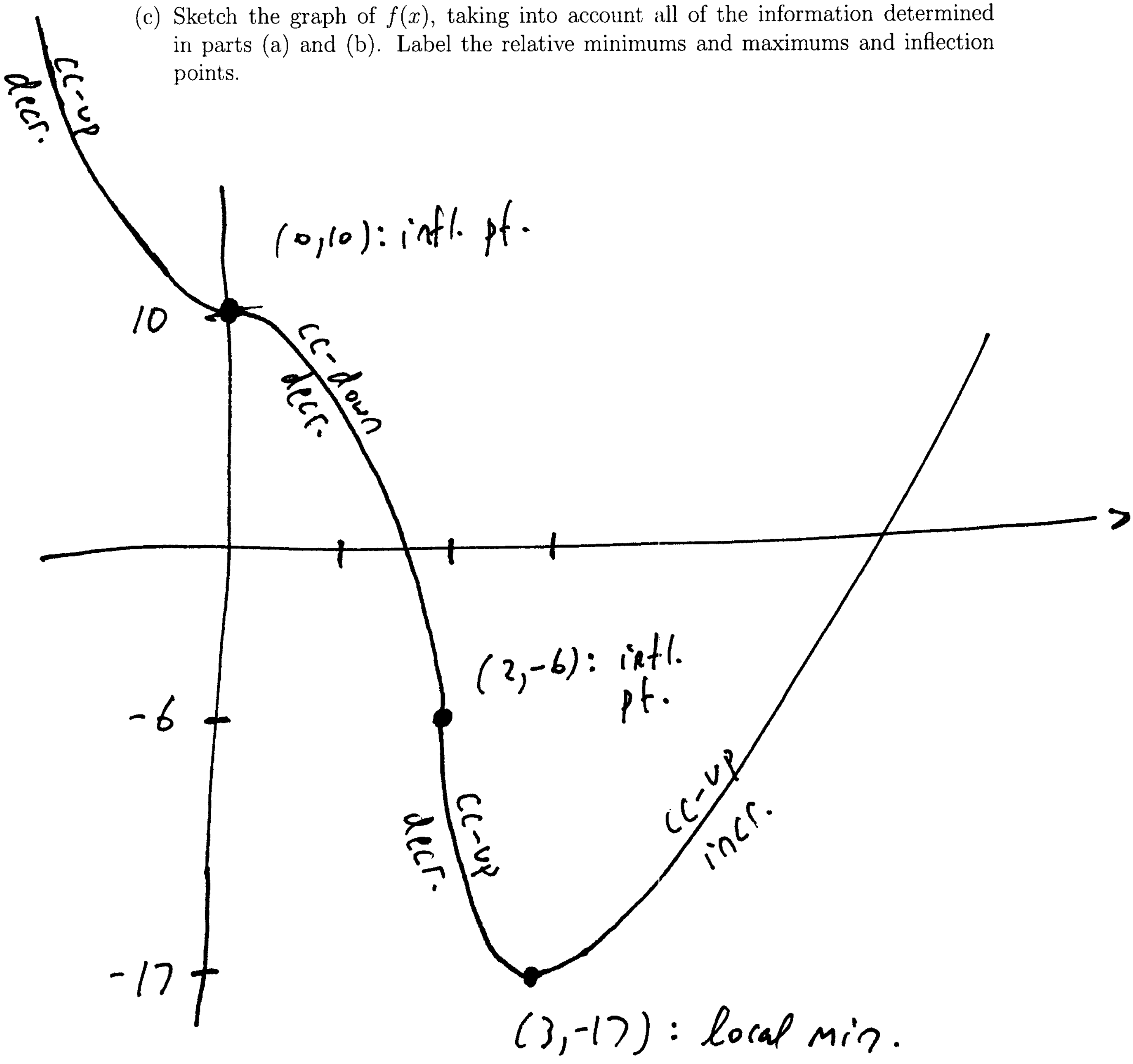
$\therefore$ Infl. pt. @ $(0,10)$

Infl. pt. @ $x=2$

y-coor: $f(2) = (2)^4 - 4(2)^3 + 10$
$= -6$

$\therefore$ Infl. pt. @ $(2,-6)$

(c) Sketch the graph of $f(x)$, taking into account all of the information determined in parts (a) and (b). Label the relative minimums and maximums and inflection points.



(d) How many roots does $f(x)$ have, i.e. for how many real numbers c does $f(c) = 0$?

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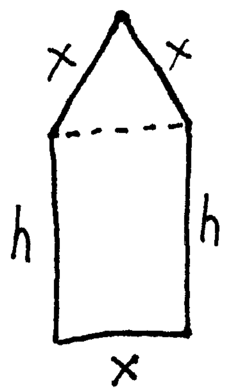
3. (28 points) Wally's Wonderful Windows is a store which sells windows which have the shape of a rectangle surmounted by an equilateral triangle. (Thus the length of each side of the triangle is equal to the width of the rectangle.)

(a) What are the dimensions of such a window with perimeter 50 inches and maximum possible area? (Hint: The area of an equilateral triangle with side length x is $(\sqrt{3}/2)x^2$.)



Maximize: $A = xh + \frac{\sqrt{3}}{2}x^2$

Constraint: $3x + 2h = 50$

$\therefore h = \frac{50-3x}{2} = 25 - \frac{3}{2}x$

$$\therefore A(x) = x\left(25 - \tfrac{3}{2}x\right) + \tfrac{\sqrt{3}}{2}x^2, \quad x \text{ in } \left[0, \tfrac{50}{3}\right]$$

$$A(x) = 25x - \tfrac{3}{2}x^2 + \tfrac{\sqrt{3}}{2}x^2$$

Crit. points of $A(x)$:

$$A'(x) = 25 - 3x + \sqrt{3}x$$

$$25 - 3x + \sqrt{3}x = 0$$

$$25 - (3-\sqrt{3})x = 0$$

$$\boxed{x = \frac{25}{3-\sqrt{3}}}, \quad \boxed{h = 25 - \frac{3}{2}\left[\frac{25}{3-\sqrt{3}}\right]}$$

We show on next page that this value of x gives a global maximum, for $A(x)$

(b) Explain how you know that the dimensions you found in part (a) give the maximum possible area.

$A''(x) = -3+\sqrt{3}$, which is a negative #.

$\therefore A''(x) < 0$ for all x in the domain of $A(x)$

$\therefore A(x)$ is concave down over its entire domain

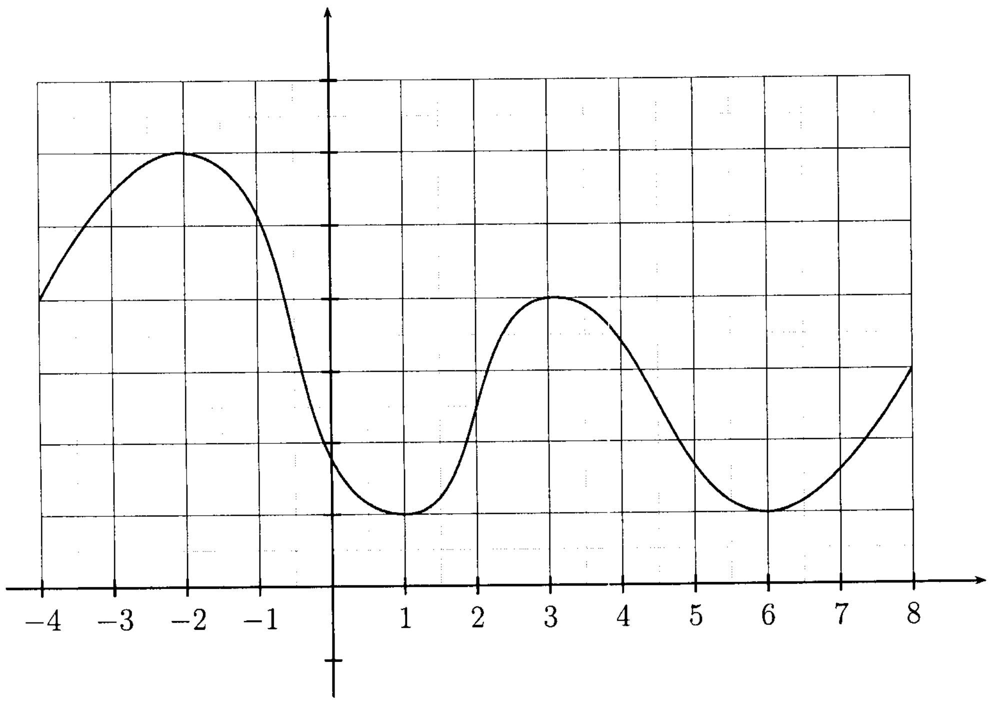
$\therefore$ the critical point @ $X = \frac{25}{3-\sqrt{3}}$ must give a global (i.e., absolute) maximum of $A(x)$.

4. (8 points) A function $g(x)$ is defined and continuous on $(-\infty, \infty)$. For each of the following, answer True or False.

(a) If $g''(c) > 0$, then $g(x)$ is increasing at $x = c$. False

(b) If $g''(c) = 0$, then $g(x)$ has an inflection point at $x = c$. False

5. (24 points) The figure below shows the graph of a function $f(x)$.



For each of the following, answer True or False.

(a) $f'(x) < 0$ if $3.5 < x < 5.5$. T

(b) $f''(x) < 0$ if $3.5 < x < 5.5$. F

(c) $f(0.5) > 0$. T

(d) $f'(0.5) > 0$. F

(e) $f''(0.5) > 0$. T

(f) $f'(-2) = 0$. T

(g) $f''(-2) = 0$. F

(h) $f(x)$ has four critical points and four inflection points, for x in $(-4, 8)$. F

(only three infl. pts.)